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ELECTRICAL MEASUREMENTS

PART I

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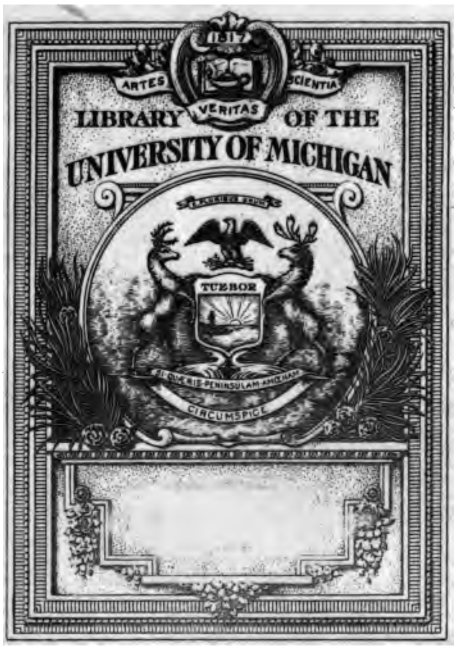

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ELECTRICAL MEASUREMENTS

PART I—ELEMENTARY

INSTRUCTION PAPER

PREPARED BY

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ELECTRICAL MEASUREMENTS

PART I—ELEMENTARY

SYSTEMS OF UNITS

Physical quantities are measured in terms of quantities called units. These units, as a rule, are related to one another and form systems; as, for example, the *British* system and the *C. G. S.* system.

Fundamental Units. The arbitrarily chosen units of a system are called *fundamental* in distinction to the related units depending on them, which are called *derived* units. The C. G. S. system, universally used in electrical measurements, takes its name from three of its fundamental units—the *centimeter,* the *gram,* and the *second of mean solar time.* Besides the three units from which it takes its name, the C. G. S. system includes other fundamental units; for example, the *degree centigrade,* the *calorie,* and the *unit magnetic pole.* Whenever the arbitrary choice of a property of a substance enters into the choice of a unit, the unit itself becomes fundamental. Thus the calorie depends on the thermal capacity of water; the unit magnetic pole depends on the magnetic property of air, etc.

Derived Units. Geometrical units, such as area and volume, are derived from the unit of length. That is, areas are measured in square centimeters, and volumes in cubic centimeters, involving units of the second and third degree with reference to the unit of length. We say that an area has a dimension of 2 and a volume of 3 in terms of a length. Put algebraically, an area may be expressed as L^2, and a volume as L^3 in terms of a length L. In mechanics we use derived units depending on length L, mass M, and time T. Thus velocity, which may be measured by the ratio of length and time, has as dimensions LT^{-1}, and acceleration LT^{-2}. Force is more complicated and may be defined in terms of the acceleration of a mass. The dimensions of force are then LMT^{-2}. The C. G. S. unit of force is called the *dyne.* Work and energy may be measured in terms of force exerted through space, and the unit, equal to one dyne acting through one

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centimeter, is called the *erg*. The dimensions of the erg are $L^2 M T^{-2}$. In the same way power (time rate of doing work) may be expressed in ergs per second. This unit of power is so small that for practical purposes we use the *watt* which is 10,000,000 ergs per second. Even the watt is small and so we frequently use the *kilowatt* (one thousand watts) for measurement of power. As we shall see later, the watt is used also for the measurement of power for electric circuits. Besides the C. G. S. units we use many units which are multiples or submultiples and so are related. For example, we use the meter (100 centimeters) and the kilometer (100,000 centimeters) and the millimeter (0.1 centimeter). Evidently the meter was intended to be the fundamental unit, the centimeter and the millimeter submultiples, and the kilometer a multiple; but in the C. G. S. system the meter becomes a multiple of the fundamental unit.

In electrical measurements the unit of resistance—the *ohm*—is practically taken as 1,000,000,000 C. G. S. units; the unit of electromotive force (e. m. f.)—the *volt*—is taken as 100,000,000 C. G. S. units; and the unit of current—the *ampere*—is taken as 0.1 C. G. S. unit. These units were originally recommended by a committee of the British Association for the advancement of science in 1873, and were internationally adopted at Paris in 1881. The watt is the practical unit of power and is equal to an e. m. f. of one volt multiplied by a current of one ampere. If the current is constant the product of current and e. m. f. gives the power. If the current is not constant, the average product of current and e. m. f. gives the average power. As we shall see later in the case of alternating currents, the readings of alternating-current voltmeters and ammeters cannot be multiplied together to get the power; but an instrument called a *wattmeter* must be used. The wattmeter gives the correct result. The watt is 10,000,000, *i. e.*, 10^7 C. G. S. units.

The unit of charge (or quantity)—the *coulomb*—is the quantity of electricity equal to a flow of one ampere for one second. The coulomb is 0.1 C. G. S. unit. The *farad* is the unit of capacity. A condenser has one farad capacity if it can store one coulomb with a potential difference of one volt at its terminals. Potential difference, like e. m. f., is practically measured in volts. At higher potential differences a condenser takes a proportionately higher charge. The farad is a very large capacity and condensers are practically rated in microfarads, *i. e.*, in millionths of a farad. The *henry* is the unit of inductance. When a current is started in a coil of wire a magnetic field is produced. This requires more e. m. f. than to maintain the current when once started. If the coil requires one volt more to increase the current at the rate of one ampere per second than to maintain it, we say the inductance of the coil is one henry. The henry is 1,000,000,000, *i. e.*, 10^9 C. G. S. units. These practical units are all related, as is seen above, to the C. G. S. units by factors, of powers of 10. There are other units in the electro-magnetic system for which the reader is referred to more advanced works.

Relation of C. G. S. to British Units. To reduce British to C. G. S. units and *vice versâ*, we make use of the relations between them. One inch equals 2.54 centimeters; one pound mass equals 453.59 grams mass; and a like relation between pounds weight (force) and grams weight. The second of mean solar time is the same in both systems. It should be kept in mind that for equal quantities the number of units is inversely proportional to the size of the unit.

ELECTRICAL MEASURING APPARATUS

Galvanometers. In the year 1819 Oersted discovered that a current flowing through a conductor produced an effect on a magnet. This effect is now explained by saying that lines of force surround the conductor, and that the north pole of the magnet tends to move along the lines of force in one direction and the south pole in the opposite direction. In other words the magnet, if free to move, tends to take a direction across the conductor. In the case of a long, straight wire the lines of force are circumferences of circles with the conductor at the center. The force on the magnet pole in this case falls off in proportion to the increase in the distance from the center of the conductor; *i. e.*, the force is inversely proportional to the distance. If the magnet is already in a magnetic field, such as that of the earth for instance, a current in a north and south wire above or below the magnet, tends to turn the magnet away from the

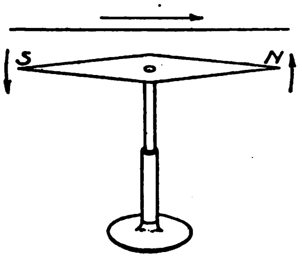

Fig. 1. Oersted's Experiment.

magnetic north and south, the tangent of the angle through which it turns being proportional to the current, Fig. 1. The effect of a single wire is small unless the current is very large.

Tangent Galvanometer. If the conductor is wound in a coil whose plane is north and south and vertical, the effect on a magnet at the center is multiplied many times, Fig. 2. Such an instrument is called a *tangent galvanometer*. If the thumb of the right hand is placed along the outside of the conductor pointing in the direction of the current, the fingers of the hand may be curled around the conductor and will point in the direction toward which the north pole of the magnet will be urged by the field produced. A similar arrangement of the left hand will indicate the direction in which the south pole will be urged.

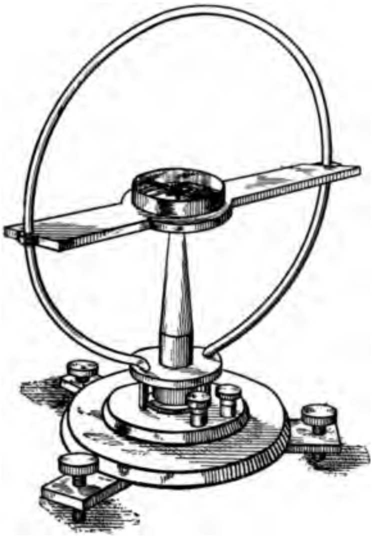

Fig. 2. Tangent Galvanometer.

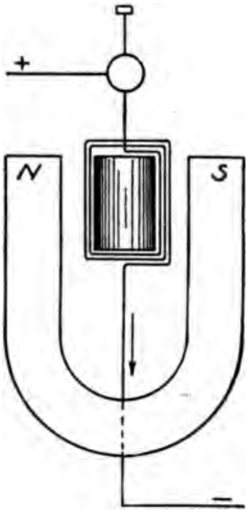

Fig. 3. Diagram of D'Arsonval Galvanometer.

D'Arsonval Galvanometer. If the magnet is fixed and the coil free to turn, the latter will turn in the reverse direction. If the magnet is of the horse-shoe type with the coil of wire between the poles a similar rule will determine the direction of motion. Galvanometers of the moving coil type were invented by D'Arsonval and Deprez, and are usually called *D'Arsonval galvanometers*, Fig. 3.

Astatic Galvanometer. An improvement may be made in the tangent galvanometer, if greater sensitiveness is desired, by mounting on the same support two magnets of nearly but not quite equal strength, care being taken to turn the poles in exactly opposite directions. This is very important. One magnet is placed at the center of the coil through which the current is sent and the other magnet is above or below the coil and influenced relatively little by the current, Fig. 4. The directive action of the earth's magnetic field is little on such a system—called *astatic*—and a small current consequently turns the system more easily from the magnetic meridian. A similar effect is produced if part of the coil is about one magnet and the rest, with reversed direction of the current, about the other magnet. Another way to produce an equivalent effect on a single, suspended magnet is to mount a powerful control magnet near by (above, below, or behind) so as to reduce to a very small amount the magnetic field due to the earth and the control magnet at the center of the coil.

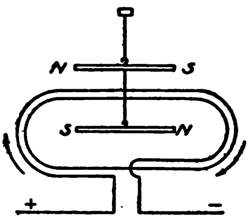

Fig. 4. Astatic System.

An extremely sensitive galvanometer may be made by combining the control magnet with the astatic system of magnets. The magnet (or system of magnets) of tangent and astatic galvanometers is suspended generally either by a fine silk or quartz fiber. The current is led into and out of the coil of the D'Arsonval galvanometer through two wires, both above in the bifilar suspension, one above and one below in the unifilar suspension.

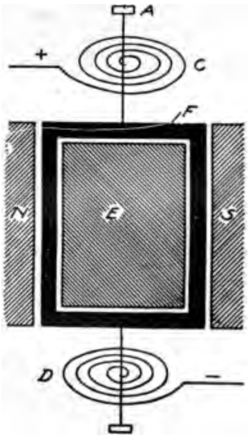

Fig. 5. Section of Suspension of Portable Galvanometer.

Less sensitive galvanometers may have their moving parts mounted on pivots or other bearings, and in such galvanometers of the D'Arsonval type the current is brought in and out through spiral springs which tend to hold the coil in its zero position, Fig. 5. Galvanometers of this type are used for ammeters—to measure amperes of current; or for voltmeters—to measure e. m. f. in volts. Such instruments are provided with some damping arrangement so that they come to rest quickly. The deflection of such galvanometers is indicated by a pointer moving on a scale. If the poles of the magnet are properly shaped the deflection may be made proportional to the current passing.

Mirror Galvanometers. Very sensitive galvanometers must be made with moving parts of little weight. It is, however, very desirable that the pointer be very long so that a large number of scale parts may correspond to small deflections. This may be accomplished by using a pencil of light rays for a pointer, as shown in Fig. 6, which illustrates the lamp-and-scale method, in which a lamp is placed behind a slit in a screen on which the scale is mounted. A concave mirror carried on the moving part of the galvanometer focuses an image of the slit at the reference point of the scale (usually the middle). When current passes, the mirror is deflected, thus deflecting the rays of light to another part of the scale. If the mirror turns through 1°, the image is deflected 2°. In place of a slit an opening of another form with cross wires may be substituted. Also if desired the lamp may be mounted at the side, and its light reflected by another mirror to the mirror on the galvanometer. In this last case it is more convenient to have the scale printed on a strip of translucent ground glass or paper, and to view the image through the glass or paper. If a telescope is substituted for the lamp, an image of the reference point of the scale may be made to coincide with the cross wire of the telescope

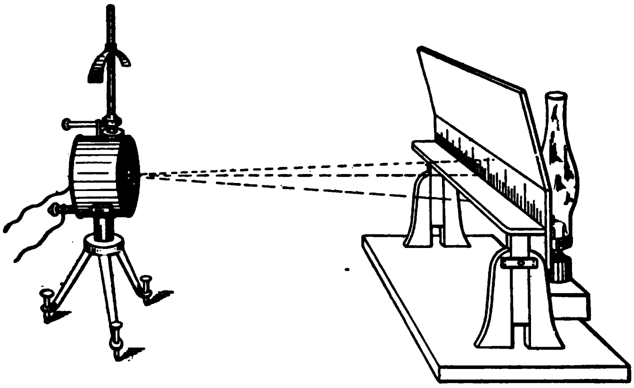

Fig. 6. Thomson Mirror Galvanometer with Lamp and Scale.

when no current is passing, and other parts of the scale will take the place of the reference point when a deflection is produced, Fig. 7. In this case a plane mirror may take the place of the concave. The telescope-and-scale method is more satisfactory for very sensitive galvanometers than the lamp-and-scale method, though the latter, usually used in a darkened room, is easier on the eyes unless an excellent galvanometer mirror and telescope are used.

Choice of Galvanometers. In choosing a galvanometer for use, it is desirable that the instrument should not be too sensitive for the experiment. As a rule the D'Arsonval galvanometer is the most satisfactory galvanometer for general use, as it is not much affected by changes in the magnetic field, even if of as great amount as produced by dynamo-electric machinery or moving of masses of iron in the neighborhood. The astatic galvanometer is, however, as a rule, far more sensitive and for certain purposes must be used.

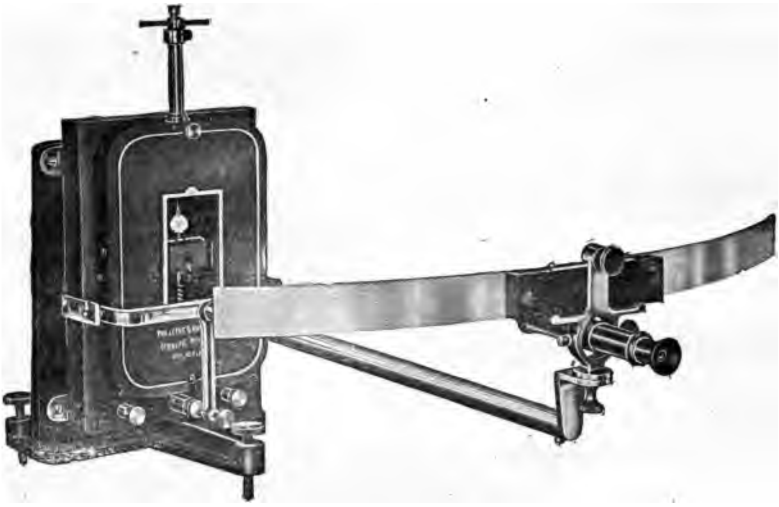

Fig. 7. Ballistic D'Arsonval with Telescope and Scale.

Use of the Control Magnet. In using astatic or other galvanometers with moving magnets, the use of the control magnet is sometimes very puzzling to beginners. The galvanometer should be set up with its coils in a north and south plane. The mirror then faces to the west (or east sometimes). The control magnet is then placed in position as far away as its support will allow and turned with its north pole to the north. The magnets and the mirror of the galvanometer, as a rule, are somewhat deflected because of the presence of the control magnet. If the latter is slightly turned in one direction, the mirror should turn in the opposite direction. As the control magnet is brought nearer, the period of swing of the mirror should increase, and the sensibility should increase in a greater proportion (as the square). If by chance the control magnet is with its south pole to the north, the mirror will turn in the same direction as the control magnet is turned, and the period of swing will decrease as the control magnet is brought nearer. Control magnets as a rule have the north pole marked in some way, so that there is no need for any mistake. When the control magnet is brought so close that the effect of the earth's field is overcome, the magnets and mirror of the galvanometer will try to turn half way around, thus turning the back of the mirror to the observer, if the construction of the galvanometer will allow. As a rule it does not pay to increase the sensitiveness of the galvanometer to the highest possible limit, as the zero reading will become very easily influenced by slight magnetic changes due to movement of small masses of iron, or the currents in neighboring conductors, or even the variation in the magnetic field due to a cloud cutting the sunlight off from the walls of a red brick laboratory, small as such an effect must be. If the galvanometer is of the astatic type, it is presupposed in the above that the support for the control magnet is arranged to weaken the field of the stronger magnet of the astatic pair more than it does the field of the weaker magnet. In some poorly adjusted galvanometers, the control magnet may produce the contrary result, and it may be necessary to make appropriate allowance. If the magnets of the astatic galvanometer take an east and west position before the control magnet is put on, it is evident that the magnets of the astatic pair are not exactly in opposite directions and that the result is a magnetic system having its effective or resultant north pole about half way between the north poles, and its resultant south pole about half way between the south poles of the two magnets. The line joining these resultant poles lies in the magnetic meridian and the magnets of the astatic pair lie nearly east and west. To correct this error in adjustment is a very delicate matter and should not be attempted by the novice.

Ballistic Galvanometer. When a charge condenser is discharged through a circuit containing a galvanometer, the galvanometer deflects. The period of swing should be long enough for practically the whole charge to pass during the early part of the swing. If the galvanometer has a short period, the return swing may begin before the discharge is complete. It may be assumed that the first deflection is a measure of the quantity discharged; but it is evident that this is an error if the discharge is slow in comparison with the time occupied by the deflection. To be on the safe side the period of swing should be large. Galvanometers which are suitable for measuring discharges are called *ballistic*. Depending on circumstances, their period may be between, say, five and twenty seconds for the complete swing. The D'Arsonval galvanometer may be made with high enough period and sensibility to give satisfaction as a ballistic instrument; but for extreme sensibility an instrument of the astatic type is more generally used. The D'Arsonval galvanometer is more nearly free from the drift of the reference point, which is due mostly to varying magnetic field and somewhat to elastic fatigue or subpermanent set in the suspension. Freedom from drift is very important, as the deflection is uncertain in proportion as the reference point is in doubt.

Damping of Vibrations. The motion of the moving system of a galvanometer may be impeded by damping. This may be accomplished by mounting vanes on the system so that the air in an enclosed chamber impedes the motion, or by electromagnetic damping produced by eddy currents induced in metal moving in a strong magnetic field. In D'Arsonval galvanometers if the coil is wound on a metal frame, currents will be induced in the frame while the coil is in motion. Such damping ensures a speedy coming to rest after a deflection and is very helpful, especially in ballistic galvanometers where certainty of zero is important. It is evident that any damping reduces the sensibility of a galvanometer. Some galvanometers are provided with so much damping that on the return swing the system does not swing past the zero or reference point. Galvanometers without a period of complete vibration are said to be *aperiodic* (the *a* denoting without). As a rule galvanometers have a complete period, that is, they are damped less than the aperiodic galvanometer. The effect of damping is to shorten the time and amplitude of the outward part of the swing (though it lengthens the complete period), and to this extent damping is objectionable. There are, however, counter-balancing advantages and so for most purposes some damping is considered wise.

Plunger Type Instruments If in place of the magnet of a galvanometer, some soft iron is substituted in such a position that the action of the current is to magnetize the soft iron and to draw it into a stronger part of the magnetic field, we have a *current indicator* of the plunger type. The coil frequently takes the form of a solenoid and the soft iron that of a rod which is drawn by the action of the current into the solenoid. The restraining force may be gravitational or that of a spring. Such an instrument is shown in Fig. 8. It is evident that the direction of the deflection does not depend on the direction of the current. In fact, plunger type instruments may be used to measure alternating currents. There are many possible variations of this type of instrument. As the iron has a certain amount of residual magnetization, the deflection with smaller following large currents is more than would have been produced by the same current following a smaller one. For this reason the plunger type of instrument is less reliable than the usual types of galvanometers. The scale is usually of unequal divisions as the pull increases more rapidly than the current.

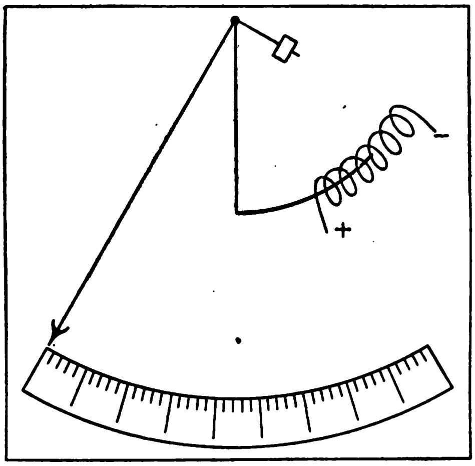

Fig. 8. Diagram of Plunger Instrument.

Electrodynamometers. If the magnet of a galvanometer is replaced by a coil through which the current passes in series with the other coil, we have what is known as an *electrodynamometer*. As in the case of the plunger type instruments, the electrodynamometer deflects in the same direction for all currents unless disturbed by being placed in a magnetic field of outside origin. It is desirable to set up an electrodynamometer with the moving coil (or coils, if more than one) with its axis (or their axes) along the magnetic meridian.

The disturbing effect of a permanent field is negligible when the electrodynamometer is used to measure alternating currents. For direct currents, the action of the outside field is eliminated by reversing the connections. The deflection is approximately proportional to the square of the current. For the best types of electrodynamometers the suspended coil is brought back to its zero position by twisting a torsion head which operates through a spiral spring on the suspended coil. The current in this type of instrument is proportional to the square root of the reading of the torsion head necessary to restore the moving coil to its zero position. A direct current producing the same deflection as an alternating current is said to be the effective value of the alternating current. Fig. 9 illustrates the usual type of electrodynamometer. Fig. 10 illustrates another form invented by Lord Kelvin and called a *Kelvin balance*. The figure shows the connections viewed from the back of the balance. The fixed coil is subdivided into four parts *B*, and the moving coil into two parts *A*, placed symmetrically between the parts of *B*. The parts of *A* are supported on opposite arms of a balance and the balance is restored to its zero position by displacing a weight along the beam.

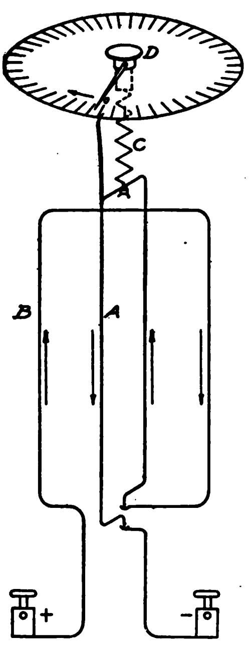

Fig. 9. Electrodynamometer Diagram.

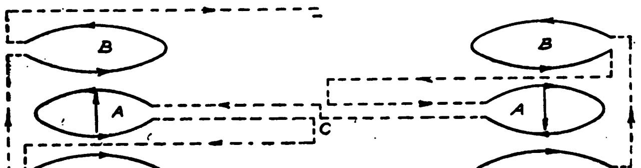

Fig. 10. Diagram of Coils in Kelvin Balance.

The action of the current is thus weighed, and the square of the current is proportional to the distance the weight is moved along the beam. The beam is divided accurately into equal parts and it is possible to obtain the reading with a high degree of accuracy. The current is proportional to the square root of the reading. The effect of dividing the two parts of A is to free the instrument from the disturbing effect of the earth's magnetic field or any other stray field of fairly uniform intensity.

Electrometers. Electrometers depend on the attraction between electrostatic charges of opposite signs. The only electrometer which we shall describe is the *electrostatic voltmeter* which consists of fixed and movable metallic parts of relatively large surface. These surfaces may be plane or curved. The terminals are connected, as a rule, one to the fixed part and the other to the movable part—the vane. These parts take charges proportional to the potential difference between them —e. m. f. applied—and a certain attraction results therefrom. If the vane is allowed to move, the electrostatic capacity of the combination increases somewhat, thus increasing the amount of the charges and the attractive force. If it is desired, the vane may be brought back to its zero position by some counter force. As a rule electrostatic instruments are allowed to deflect and are calibrated by comparison with other forms of voltmeters. More complicated forms of electrostatic instruments may have two sets of fixed surfaces and a movable vane. In some cases a battery of cells of known e. m. f. may be used to charge the fixed surfaces, and the e. m. f. to be measured may be applied between one fixed surface and the vane. Electrostatic voltmeters are generally used to measure high electromotive forces. Fig. 11 shows an electrostatic voltmeter of an old type, which shows

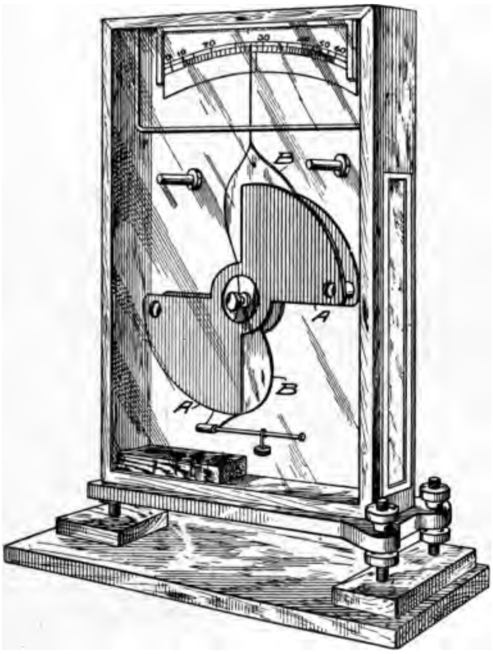

Fig. 11. Electrostatic Voltmeter.

the general scheme more clearly than better and more complicated electrometers.

Hot Wire Instruments. If current passes through a wire, a heating effect results and the wire lengthens because of its rise in temperature. If a pointer is held in a position of equilibrium between turning moments produced by two wires pulling on opposite arms of a lever, the heating of one of these wires by an electric current will produce a change in the position of equilibrium. It is evident that change in the temperature of the room affects both wires alike and produces no change in the zero position. The deflection of a hot wire instrument is dependent on the square of the current (as the heating is proportional to the square of the current). For this reason the hot wire instrument deflects in the same direction for currents in either direction and for alternating currents as well. As the effective value of an alternating current is equal to the square root of the mean square, it is evident that a hot wire instrument calibrated by direct currents, will give proper readings for alternating currents also. Hot wire instruments are made use of as ammeters (low resistance) and voltmeters (high resistance). As a rule hot wire instruments are used for alternating currents. They are usually less accurate than electrodynamometers of the best types.

Wattmeters. We have seen that an electrodynamometer has a turning moment proportional to the square of the current passing through it. If the current passing through the fixed coil is different from that passing through the movable coil, the turning moment will be proportional to the product of these currents. If the power delivered to a line is to be measured, the average product of the volts and amperes gives the result in watts. The current delivered from the line to the load may be passed through one coil (usually of low resistance) whose terminals are A and B, Fig. 12, and the e. m. f. may be applied at the terminals of the other coil (usually of high resistance) whose terminals are a and b, and produce a second current proportional to this e. m. f. In order to avoid measuring the effect of the pressure current it is led backward through coil F shown in dotted line, thus subtracting its effect. The instrument may be calibrated to read watts. As a rule it is easier to make the second coil of moderate resistance and to insert a non-inductively wound high resistance coil R in series. If the wattmeter is to be calibrated by the use of current and e. m. f. in separate circuits, the terminals *i* and *b* are used. The resistance of *S* is equal to that of *F*. The currents in both coils will reverse if the e. m. f. is reversed, but the deflection will be unchanged. The average power of a varying current equals the average product of current and e. m. f.; consequently a wattmeter calibrated with direct currents gives correct results for alternating currents. If the current and e. m. f. are alternating, the mean product will in general be less than the product of the effective values of current and e. m. f. (as measured by A. C. voltmeters and ammeters); consequently when dealing with alternating current and e. m. f. the product must be multiplied by a factor, called the *power factor*, which is usually less than unity, if the correct value is to be computed from ammeter and voltmeter readings. As a rule the power factor is found by dividing the watts as measured by a wattmeter by the product of volts and amperes.

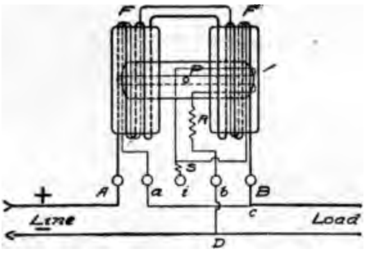

Fig. 12. Diagram of Wattmeter.

Recording Voltmeters and Ammeters. If any of the voltmeters or ammeters described above are arranged with a pen which traces a line on a disk or roll of paper drawn by clockwork past the pen, the instrument will record the variations of e. m. f. or current. There are several good types of recording voltmeters and ammeters on the market. A recording ammeter is shown in Fig. 13.

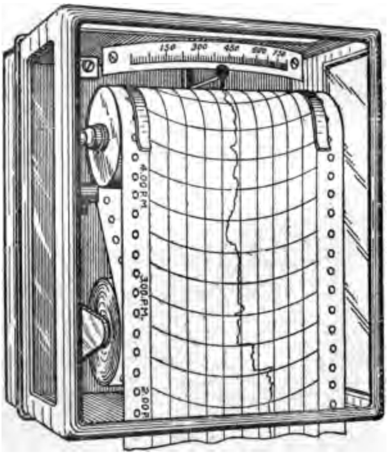

Fig. 13. Recording Ammeter.

Integrating Watt-Hour Meters. Integrating meters show the total consumption of the thing to be measured; for example, integrating gas meters show the consumption of gas in cubic feet. In the same way an integrating watt-hour meter (commonly, though inexactly, called an integrating wattmeter) shows the consumption of energy in watt-hours. Such an instrument is shown in Fig. 14. The instrument is essentially an electric motor geared to a train of wheels moving hands over dials. The speed of the motor is proportional to the power in watts, and the product of the average power and the time in hours (that is, watt-hours) is indicated by the change in the position of the hands on the dial since the last reading. To give correct readings the driving motor must be designed for the circuit on which it is used. The essential factors of the circuit are the e. m. f., maximum current, whether direct or alternating current is used, etc. In a three-wire system a single meter may be designed to measure the power of the two or three circuits involved.

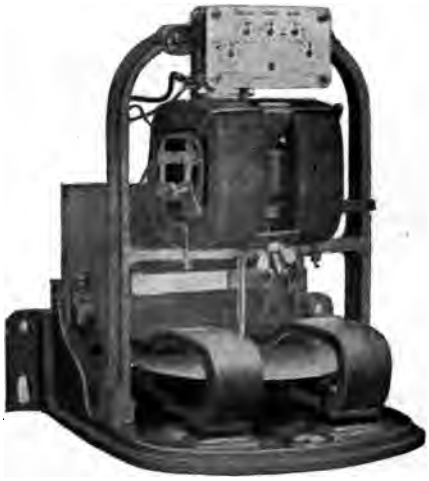

Fig. 14. Watt-Hour Meter.

Integrating Ampere-Hour Meters. An integrating ampere-hour meter (commonly called integrating ammeter) is similar to the watt-hour meter. It is used generally in connection with storage batteries to keep account of the charge and discharge. It is of little general use.

Rheostats and Resistance Coils. The word rheostat means an apparatus for stopping the current. In actual fact it does not wholly stop the current, but only reduces it to a desired extent. Every material interposes some resistance to the flow of an electric current. Substances interposing extremely high resistance are known as insulators, and those interposing relatively little resistance, as conductors. Metals, as a rule, are the best conductors. The metals most used commercially for electrical transmission are copper, aluminum, and iron (or steel). Alloys in general have much higher resistance than the metals of which they are composed. Carbon and solutions of various salts have much higher resistance than metals. Rheostats may be made of any of these materials, but those most generally used are steel wire or sheets, German silver wire or other alloys, carbon rods or plates, and solutions in tanks in which metal plates are immersed, the metal plates being connected to the terminals of the circuit. Such metal plates are known as electrodes. This last arrangement is usually called a *water rheostat.* Pure water has a very high resistance and is never used in water rheostats, but the resistance may be reduced as desired by dissolving salt in the water. The metal plates are usually arranged so that one electrode may be brought nearer the other when it is desired to increase the current. When the word rheostat is used, it is generally understood that the resistance is not exactly known. A rheostat is shown in Fig. 15.

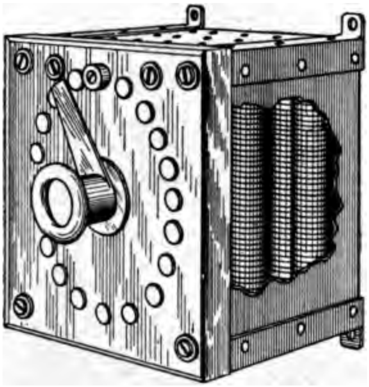

Fig. 15. Rheostat for Varying the Current in Any Circuit.

When it is desired that the resistance have a certain exact value, metals are the only practical materials to use. Coils of wire exactly adjusted are called *resistance coils.* They are adjusted to certain values in ohms. For very low resistances, *e. g.*, small fractions of an ohm, metal strips may be used. As most pure metals increase their resistance with increase of temperature by about 0.4% per degree centigrade, resistance coils are almost always made of certain alloys which change little in resistance with change in temperature. One alloy in particular, manganin, changes so little in resistance with change in temperature that it is usually chosen for standard *resistance* coils. Figs. 16 and 17 show standard resistances in the form of a coil and a strip. As mentioned above alloys have relatively high resistance and for this reason also the alloy manganin is preferable to any pure metal for resistances.

Lamp Rheostats. A very convenient form of carbon rheostat is a bank of incandescent lamps. The usual 16 c. p. lamp for a 110-volt circuit has a resistance when hot of about 220 ohms. Its resistance when cold is about twice as much. Carbon and solutions, unlike metals, are better conductors when hot than cold. It is evident that incandescent lamps, because of their change in resistance from cold to hot, are not suitable for standard resistances. If two lamps are arranged in series, *i. e.*, if the current is made to pass through one after the other, the resistance of the combination is twice that of a single lamp. On the other hand if the lamps are connected in parallel, *i. e.*, if the current divides between them, the resistance of the combination is only half of that of a single lamp. This result is evident as the same e. m. f. produces twice as much current in two lamps as in a single one. In the same way ten lamps in parallel have a combined resistance only one-tenth as much as a single lamp.

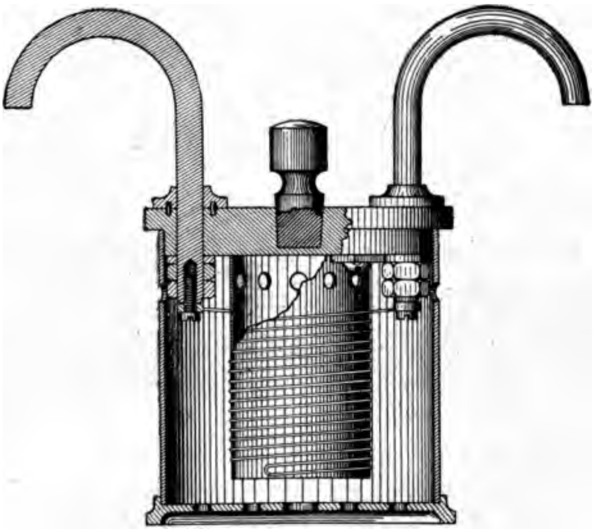

Fig. 16. A Standard Resistance Coil.

Multiplying Power of Shunts. The word shunt is the British name for a side track (or as we would call it, switch) on a railway. Any electrical side path is called a shunt. If the current has two or more paths in parallel offered to it, the current divides in inverse proportion to the resistance or, as more simply expressed, in direct proportion to the conductivity of the various paths. Thus if a galvanometer has a shunt across its terminals whose resistance is one-ninth of that of the galvanometer, nine times as much current will go through the shunt as through the galvanometer and consequently only one-tenth of the current will go through the latter. The total current is then ten times the current through the galvanometer and we

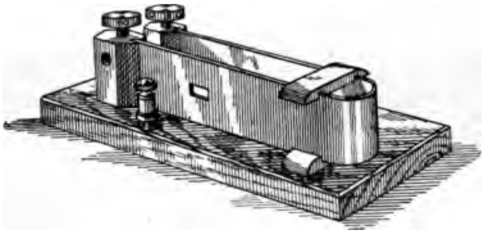

Fig. 17. A Standard Resistance Strip.

say that the multiplying power of the shunt is ten. If the galvanometer has a shunt of one ninety-ninth of the former's resistance, one one-hundredth of the current will pass through the galvanometer and the multiplying power of the shunt will be one hundred. In general, if the resistance of the shunt is $\frac{1}{m-1}$ of that of the galvanometer, the multiplying power of the shunt will be m. The evident effect of the shunt is to reduce the resistance of the galvanometer circuit to $\frac{1}{m}$ of its former value, *i. e.*, to $\frac{1}{m}$ of the resistance of the galvanometer itself; the resulting fall of potential over the galvanometer and shunt is, therefore, only $\frac{1}{m}$ as much as if the shunt were not there. Galvanometers are provided by their makers, if desired, with shunts having a multiplying power of 10, 100, and 1,000, marked to go with the particular galvanometer. It is evident that the usual shunt cannot be used with other galvanometers without recalculation of its multiplying power, which under such circumstances would probably be some inconvenient number.

Professor Ayrton has devised a form of shunt box with extra resistance which is automatically connected in series in proper amount to keep the total resistance constant but allowing only $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$ of the current to pass through the galvanometer.

OHM'S LAW

In 1827, Dr. G. S. Ohm of Berlin published a treatise, now famous, entitled *The Galvanic Circuit Investigated Mathematically*, in which he announced the fundamental law of electric circuits now known as Ohm's law. This is usually stated in the algebraic formula:

$$I = \frac{E}{R}$$

V

In words, the current (in amperes) equals the e. m. f. (in volts) divided by the resistance (in ohms). It is truly a surprising fact that the resistance of an electric circuit is a constant not dependent on the current passing. Many experimenters have tried in vain to find any inaccuracy in Ohm's law. If any two of the three quantities involved are known, the third may be found by solving the equation. Thus,

$$I = \frac{E}{R},\ R = \frac{E}{I},\ \text{and } E = I\,R$$

As will be seen later, one of the most convenient methods of measuring low resistances, as of a dynamo armature, is a simple application of Ohm's law.

MEASUREMENT OF RESISTANCE

Resistance Boxes. Measurement of resistance is made by comparison with certain standards of known resistance, the different methods of measurement varying to a great degree. The standard resistance coils are made of such alloys as manganin—an alloy of manganese copper and nickel—which has a high specific resistance and changes its resistance with rise in temperature to a much less extent than other metals. It is of course desirable that this change should be as small as possible. The size and length of the coils are such that they have resistances of a definite number of ohms at a certain temperature. The coils are insulated with silk or paraffined cotton and are very carefully wound. Each wire is doubled on itself before being coiled up, and then wound as shown at *A* and *B* in Fig. 18; or, as is sometimes preferred, the wire may be wound single in layers, the direction of winding being reversed for alternate layers. Inductance and capacity effects are by these means reduced to a minimum. The ends of the coils are soldered to brass pieces as *C*, *D*, *E*. Removable conical plugs *F* and *G* of brass are made to fit accurately between the brass pieces. When these are inserted as shown, the coils will be short circuited and a current will pass directly through *C*, *F*, *D*, *G*, and *E* without going through the coils. If *F* is withdrawn the coil *A* will then be inserted in the circuit; if *G* is also withdrawn then coils *A* and *B* will both be inserted, as the current cannot pass from *C* to *E* without going through the coils.

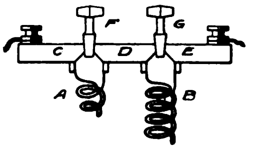

Fig. 18. Resistance Box Coils Showing Non-inductive Winding.

Resistance boxes are constructed consisting of a large number of resistance coils, and of such resistances that by withdrawing plugs varying resistances may be built up. A common form of resistance box has coils of the following ohms resistance: 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500, 1,000, 2,000. A resistance of 497 ohms could be made up by withdrawing plugs corresponding to the coils 200 + 200 + 50 + 20 + 20 + 5 + 2 = 497, or 768 by coils 500 + 200 + 50 + 10 + 5 + 2 + 1 = 768.

Resistance by Substitution. By Ohm's law the greater the resistance inserted in a circuit the less becomes the current, provided the e. m. f. remains constant. This gives us a simple although not very accurate method of measuring electrical resistance. If a battery of constant e. m. f., the unknown resistance, and a simple galvanometer are connected in series, the strength of the current passing will be indicated by the latter. Suppose the unknown resistance to be replaced by known resistances, enough resistance coils being inserted so that the deflection of the galvanometer needle is the same as when the unknown resistance was in circuit. The current will then be the same, and as the e. m. f. remains unchanged, the resistances must be equal in each case. The sum of the known resistance coils inserted will then be equal to the unknown resistance.

The advantages of this method are that it is rapid, and that only crude apparatus is required, as the galvanometer and resistance box can be very simple in form. The resistance of the battery and galvanometer should be but a few ohms, otherwise small resistances cannot be measured closely. Only small currents should be used so that the error from heating may be negligible.

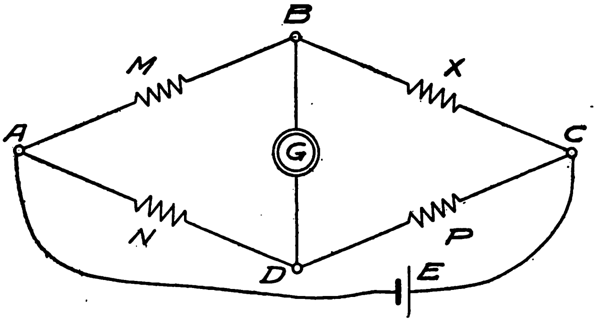

Fig. 19. Theoretical Diagram of a Wheatstone's Bridge.

Wheatstone's Bridge. All ordinary measurements of resistance are usually made by use of a Wheatstone's bridge.

The principles of this instrument will be understood from Fig. 19. There are four arms to the bridge with the resistances M, N, X, and P. From the points of junction A and C, wires connect with a battery E. A galvanometer G is connected between the junction points B and D. The current from the battery divides at A and passes through the resistances *M* and *X*, and *N* and *P*, uniting again at *C*. The fall of potential between *A* and *C* must of course be the same in amount through the resistances *M* and *X* as through *N* and *P*. If no current passes through the galvanometer then the points *B* and *D* will be at the same potential, and there will be the same fall of potential in the resistances *M* and *N*, and in the resistances *X* and *P*. Under these circumstances the ratio of the resistances of *M* to *N* will be the same as *X* to *P*, or

$$\frac{M}{N}=\frac{X}{P}$$

If *M*, *N*, and *P* are known resistances, the resistance of *X* is readily found by the formula,

$$X=\frac{M}{N}\times P$$

The method of using the bridge will be better understood from Fig. 20. The bridge arm *M* has coils of 1, 10, 100 ohms resistance, and arm *N*, coils 10, 100, 1,000. The series of coils *P* for obtaining a balance usually has resistances of 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500, 1,000, 2,000 ohms, but coils up to 100 only are shown. There is a key *K* in the galvanometer circuit and a key *H* in the battery circuit. The battery key *H* should always be closed *before* the galvanometer key *K*, and should be kept closed until after *K* is opened. This not only insures steadiness in all currents when the galvanometer circuit is closed, but also protects the galvanometer from self-induction currents which would occur if the battery circuit were closed after that of the galvanometer. A double successive contact key, Fig. 21, may with advantage be substituted for the two single keys, thus insuring that the battery and galvanometer branches will be closed and opened in the proper se-

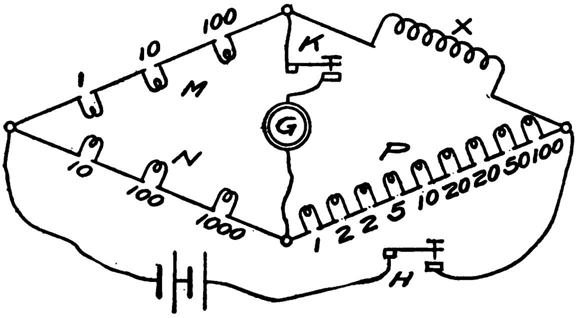

Fig. 20. Diagram Showing Method of Making Bridge Measurements.

quence. A reflecting galvanometer is used for accurate measurement

In making a measurement of an unknown resistance it is first necessary to gain a knowledge of its approximate resistance. For this purpose the 100-ohm plug is withdrawn from both arms *M* and *N*, the unknown resistance being connected at *X*. The ratios of *M* to *N* will then be unity, and hence for a balance the number of ohms required in the resistance coils *P* will be the same as the resistance *X*. The 1,000-ohm plug in *P* should first be drawn and the keys depressed in their proper order for an instant only. The galvanometer needle or mirror, as seen by the light reflected on the scale, is deflected—say to the right, and the resistance is probably too large. The plug is replaced and the 1-ohm coil withdrawn. On depressing the keys suppose the spot of light is deflected to the left. Then the 1 ohm is too small and the 1,000 ohms too large; also in this case deflections to the right mean that the resistance inserted is too large, and to the left, that the resistance inserted is too small. The 1-ohm plug is now replaced, and 500, 200, etc., are successively tried until it is found that 12 ohms is too large and 11 ohms too small, that is, the unknown resistance is between 11 and 12 ohms.

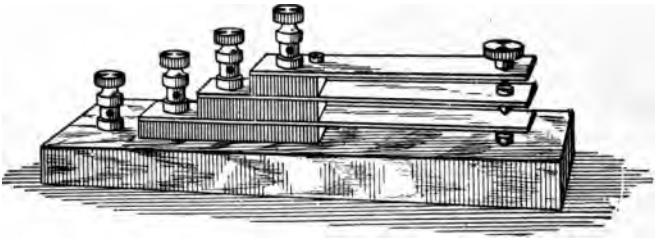

Fig. 24. 4-Point Contact Key.

Suppose that it is desired to find the correct value of the unknown resistance to the second place of decimals. The ratio of the arms *M* to *N* must then be changed so that the resistance coils *P* will have a value of between 1,100 and 1,200 ohms when a balance is obtained. The ratio of *X* to *P* will then be 11 to 1,100 approximately, or about 1 to 100. To obtain a balance the ratio of the arms *M* to *N* must also be 1 to 100. Hence the 100-ohm plugs first withdrawn are replaced and the 10-ohm plug withdrawn from *M* and the 1,000-ohm plug from *N* giving the required ratio. The same ratio could be obtained by withdrawing the 1-ohm plug in *M* and the 100-ohm plug in *N*.

The bridge is now arranged for the final measurement. As the resistance in P will now be over 1,100 ohms, the 1,000- and 100-ohm plugs are first removed. Suppose the 50-ohm plug to be also removed, and a deflection to the right shows that this is too great. The plug is replaced and 20 withdrawn, which proves to be too small. The next twenty plug is also withdrawn and a deflection to the left shows the resistance to be still too small. The 5-, 2-, and 2-ohm plugs are successively withdrawn, the last two ohms proving to be too great. This is replaced and the 1-ohm plug withdrawn, and suppose no deflection is then obtained. The total number of ohms in P is now $1{,}000 + 100 + 20 + 20 + 5 + 2 + 1 = 1{,}148$. The value of X is therefore $\frac{10}{1000} \times 1{,}148 = 11.48$ ohms.

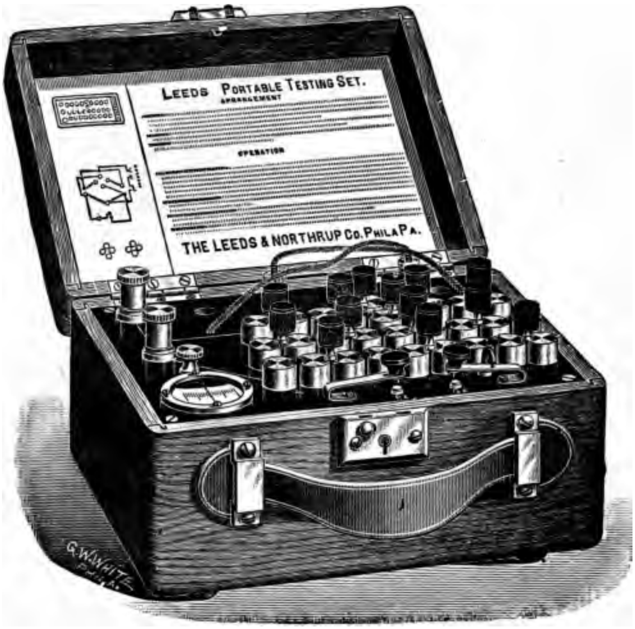
LEEDS PORTABLE TESTING SET.
ARRANGEMENT
OPERATION
THE LEEDS & NORTHRUP CO. PHILA PA.
G.W.WHITE PHILA.

Fig. 22. Portable Testing Set.

The above example illustrates the general method of using the bridge. Usually the resistance to be measured is known approximately and the required ratio between M and N can be determined without making a preliminary measurement. The possible changes in the ratio between M and N gives the bridge a great range of measurement. When M is 1 and N is 1,000 ohms, measurements of resistance as small as .001 ohm may be made. Bridges are usually arranged with a reversing key so that M and N may be interchanged, hence M could be 1,000 and N 1, and measurements of resistance as high as 4,110,000 ohms could be made with the bridge we have considered.

Portable Testing Set. There are many different varieties of bridges and their form always differs from that of the diagrams in Figs. 19 and 21. A portable testing set including Wheatstone's bridge, galvanometer, battery, and keys, is illustrated in Fig. 22. The rheostat of the bridge is made up of coils, 16 in number, of denominations 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1,000, 2,000, 3,000, 4,000 ohms—11,110 ohms in all. Bridge coils are 1, 10, and 100 on one side and 10, 100, and 1,000 on the other. A reversing key admits of any ratio being obtained in either direction so that the range of the set is from .001 to 11,110,000 ohms. It is, however, impossible to construct a portable galvanometer of sufficient sensitiveness for these measurements, and the actual limits are from .001 ohm to 300,000 or 400,000 ohms.

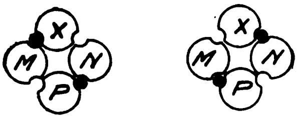

Fig. 23. Reversing Keys.

The reversing key, shown in Fig. 23, consists of the blocks M, N, P, and X and two plugs which must both lie on one diagonal or the other. The blocks are connected with the resistances indicated by their letters. In the left-hand figure M is connected with X and N with P, and the bridge arms have the relation

$$\frac{M}{N} = \frac{X}{P}, \text{ or } X = \frac{M}{N} \times P$$

In the right-hand figure M is connected with P and N with X, the bridge arms then having the relation

$$\frac{M}{N} = \frac{P}{X}, \text{ or } X = \frac{N}{M} \times P$$

The advantages of having a reversing key in the bridge arms are: the increase in range obtained, six coils being made to do the work of eight, and also that any error in the initial adjustment of the bridge arms can be detected by having the two arms equal, balancing and reversing. Unless the resistance of the coils inserted in M and N are exactly equal, the system will be unbalanced after reversing.

The galvanometer, the needle and scale of which are shown at the left in Fig. 22, is of the D'Arsonval type, and the coil is mounted in jewels. As this galvanometer is not affected by external magnetic fields or electric currents, it is suitable for dynamo or shop testing. The key for the galvanometer circuit is shown in front at the right.

The battery is made up of chloride of silver cells mounted in the bottom of the box. The cells will last a number of months even with daily use. Flexible connecting cords, running from the cells, have their terminal sockets combined with small binding posts so that connection may be made to an extra battery or other source of e. m. f. if desired. The left-hand key controls the battery circuit.

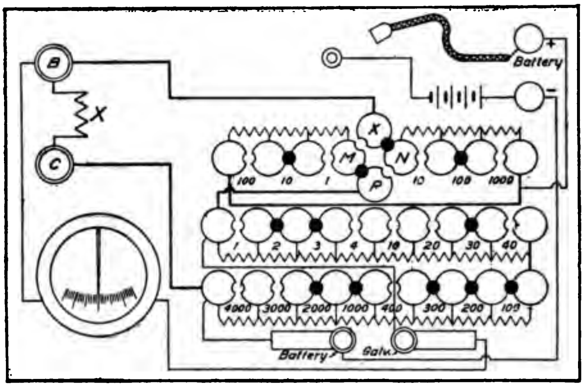

Fig. 24. Diagram of a Testing Set.

A plan of the connections of this testing set is shown in Fig. 24. The two lower rows of coils (marked 1 to 4,000) are connected beneath the top at the right by a heavy copper rod and constitute the rheostat arm, or what corresponds to *P* in the formula. By withdrawing the proper plugs in these rows any number of ohms from 1 to 11,110 may be obtained. The upper row of coils consists of the two bridge arms, *M* at the left and *N* at the right, with the reversing key between them. The two extremes of the upper row are joined by a heavy copper connection and correspond to the point *A* in Fig. 19. The upper block *X* of the reversing key is connected with the binding post *B*, the block *P* is joined to the left of the middle row of coils while the other end of the rheostat combination is connected with the binding post *C*. The resistance to be measured *X* is connected between the terminals *B* and *C*.

Example. Suppose a balance is obtained with an unknown resistance connected between *B* and *C*, when the plugs are withdrawn as shown in Fig. 24. What is the value of the unknown resistance?

Solution. The reversing key is arranged so that M is connected with P and N with X, hence

$$\frac{M}{N} = \frac{P}{X}, \text{ or } X = \frac{N}{M} \times P$$

In the figure $N = 100$, $M = 10$, and $P = 2{,}000 + 1{,}000 + 300 + 200 + 100 + 30 + 3 + 2 = 3{,}635$. Therefore

$$X = \frac{100}{10} \times 3{,}635 = 36{,}350 \text{ ohms}$$

Ans. 36,350 ohms.

Use and Care of Bridge. Before beginning a measurement it is essential that each plug be examined to see that it is firmly twisted into place, also in replacing a plug the same care should be used. A slight looseness will considerably increase the contact resistance and so introduce errors in the result. Moderate force only is needed in placing plugs. A strong person may damage the apparatus. For the same reason the plug tapers should be kept clean and the top of the bridge should be free from dust and moisture. Special care should be taken with the surfaces between adjacent blocks. The plugs should be handled only by their vulcanite tops, and care should be taken not to touch the blocks.

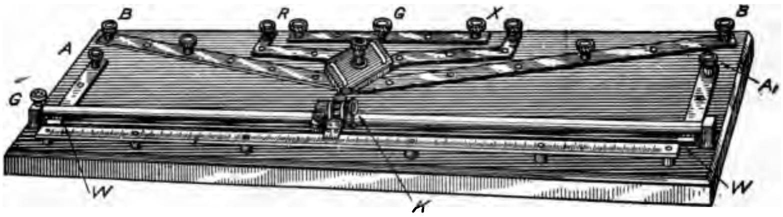

Fig. 25. Slide Wire Wheatstone's Bridge.

The plug tapers may be cleaned with a cloth moistened with alcohol and then rubbed with powdered chalk or whiting. The powder should be entirely removed with a clean cloth before the plugs are replaced. Sand paper or emery cloth should never be used to clean the plugs or bridge blocks. If there are no idle sockets for the reception of the plugs when they are withdrawn, they should be stood on end or placed on a clean surface.

Slide Wire Bridge. The simplest form of Wheatstone's bridge is the slide wire bridge. Fig. 25 illustrates the apparatus. The foundation of the bridge is a board well braced to avoid warping, which, after being well dried, is saturated with hot paraffin to make it a good insulator. The bridge wire is usually one meter long and stretched between substantial anchorages at the ends. Heavy straps of copper or brass serve as connections of negligible resistance to the other parts of the bridge. The known resistance is inserted at R and the unknown at X. Openings at A and A' are closed by heavy metal straps for the usual method of use, or in more complete methods by resistances which are, in effect, extensions of the bridge wire. The battery and its key are connected between B and B, and the galvanometer between G and G. The heavy rod back of and above the bridge wire is a support for the galvanometer key K and the index which is adjacent to the meter scale shown. The key and the index may be moved along the rod to find the balancing point. A commutator shown at the center of the apparatus serves to exchange the relative position of X and R in the arrangement. The commutator makes connection in four mercury cups. If the portion of the bridge wire to the left of the galvanometer key is a cm. long, the rest of the wire is $100-a$ long. If the commutator is arranged so that R is connected to the left end and X to the right end of the wire, when a balance is reached we have

$$\frac{R}{X} = \frac{a}{100 - a}, \text{ or } X = R\,\frac{100 - a}{a}$$

If the commutator is reversed and the new reading is a' we get

$$X = R\,\frac{a'}{100 - a'}$$

If the balancing point is near the end of the wire, it is evident that any small error in the reading and the assumption that the connections are of negligible resistance, will result in greater error in the final formula. For this reason it is well to treat the first balance as only approximate and after calculating X to take as known resistance a new value of R as nearly as possible equal to X. In this way the balancing point will be brought near the center of the wire.

For very exact comparison of two nearly equal resistances, we insert auxiliary resistances at A and A'. These are in effect extensions of the bridge wire. Call these resistances equal to A and A' cm. of the wire. When a balance is obtained at the points a and a' for the two positions of the commutator, we have

$$X = R\frac{A' + 100 - a}{A + a} \text{ and } X = R\frac{A + a'}{A' + 100 - a'}$$

While it is still of advantage under these conditions to have a and a' somewhere near 50 cm. it is no longer necessary, for with the bridge wire extended by A and A' any point of the actual bridge wire is now near the center.

If R is materially larger or smaller than X, the balancing point may be beyond the end of the actual wire, *i. e.*, in one of the extensions, and no balance can be obtained. It is necessary then to adjust R until a balancing point is found on the wire. We may then proceed with the experiment.

A variation of this method, known as the *Carey-Foster method*, is used for the comparison of two standard resistances to discover small differences in adjustment.

Example. If with the openings A and A' closed with straps of negligible resistance and a resistance of 150 ohms for R, the mean balance point comes so that $a = 68.4$ cm. and $b = 31.6$ cm., what is the value of X? Ans. 69.3 ohms.

Example. If A and A' are equivalent to 500 cm. each, and $R = 150$ ohms, and the mean balance point makes $a = 68.4$ cm., and $b = 31.6$ cm., what is the value of X? Ans. 140.29 ohms.

Low Resistance Measurement. The bridge methods described above are not suitable to use in the measurement of small resistances, for the *lead* wires (*leading in* wires) used in connecting the unknown resistance to the *bridge* may have more resistance than the unknown. The *ammeter-voltmeter method* is that most generally used. The apparatus is connected as shown in Fig. 26. The current from the battery is led through the ammeter to the unknown low resistance R. An adjustable resistance r of a rheostat may be introduced into the circuit to control the current. The actual resistance r need not be known. The fall of potential V through R is measured by the voltmeter, and the current I by the ammeter. Ohm's law then gives

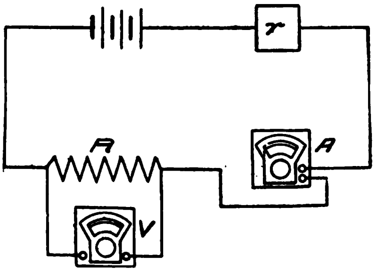

Fig. 26. Ammeter-Voltmeter Method of Low Resistance Measurement.

$$R = \frac{E}{I}$$

Ammeter-Voltmeter Method. It is evident that although the voltmeter is of very high resistance, a small current included in that measured by the ammeter passes through the voltmeter. In strictness this current, equal to $\frac{V}{\text{resistance of the voltmeter}}$, should be subtracted from the ammeter reading to get the value of I to be used in the formula. This correction is easily made, as all makers of voltmeters give the value of the resistance, usually marked on the voltmeter case; but the error resulting from neglecting the correction is generally immaterial. Instruments of suitable range should be used.

Example. The reading of the ammeter is 50 amperes, that of the voltmeter 1.5 volts; what is R? Ans. 0.03 ohm.

In the particular case chosen the ammeter had a range 0 to 75 amperes, and the voltmeter 0 to 3 volts. The resistance of the voltmeter was 300 ohms. The current through the voltmeter was $1.5 \div 300 = 0.005$ amperes. It is evident that the correction is far smaller than the probable error of reading the ammeter, and any attempt at correction would be absurd.

This method may also be used to measure the resistance of a burning incandescent lamp. In such a case the bridge method is useless as the resistance of a cold lamp is probably double its resistance when hot.

Example. The voltmeter, 0 to 150 volts range and resistance 15,000 ohms, reads 110 volts; the ammeter, 0 to 1 amperes range, reads 0.5 ampere. What is the resistance R? Ans. 220 ohms.

High Resistance Measurement. *Direct Deflection Method.* An excellent method of measuring resistances of one megohm (one million ohms) or more, is the direct deflection method. The main instruments needed are a sensitive galvanometer, usually of high resistance and fitted with appropriate shunts; some standard resistances of 100,000 ohms (0.1 megohm) or more; and a battery of relatively low resistance and constant e. m. f. (a storage battery of many cells, if available). The resistance of the galvanometer both alone and combined with its shunts must be known. That of the battery and the connections is usually neglected. The connections are shown in

Fig. 27. The known resistance R is first connected in series with the galvanometer G and the testing battery B, through a key K. Care should be taken that the insulation of the apparatus be very high. The shunt S is adjusted to give a suitable deflection of the galvanometer and from this deflection what is known as the *constant* is calculated. The value of this constant is the resistance that must be inserted in the circuit to reduce the deflection to one scale division. The value of the constant is therefore equal to the product of the total resistance, assumed to be $R + \frac{G}{m}$, the scale deflection d, and the multiplying power of the shunt m. We thus get

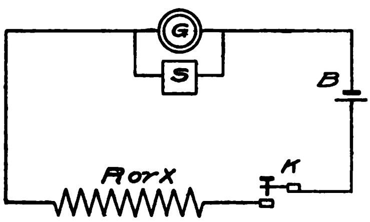

Fig. 27. Direct-Deflection Method of High Resistance Measurement.

$$\text{Constant} = (R + \frac{G}{m})\, d\, m$$

As an illustration, suppose $R = 0.1$ megohm, $G = 20{,}000$ ohms $= 0.02$ megohm, $m = 1{,}000$, and $d = 200$ divisions; then

$$\begin{aligned}\text{Constant} &= (0.1 + 0.00002) \times 200 \times 1{,}000\\ &= 20{,}004 \text{ megohms}\end{aligned}$$

This means that if the galvanometer were unshunted and the total resistance in the circuit were 20,004 megohms, a deflection of one division would result.

After the constant has been determined the known resistance R is replaced by the unknown resistance X. The galvanometer shunt is readjusted if necessary and the deflection obtained is again noted. The value of the total resistance is then found by dividing the value of the constant by the product of the deflection d_1 and the multiplying power m_1 of the shunt used. To continue our illustration suppose $d_1 = 50$ divisions, and $m_1 = 10$. The deflection, if the full current went through the galvanometer, would be $50 \times 10 = 500$ divisions. A deflection of one division is produced with a resistance of 20,004 megohms; hence a deflection of 500 divisions must correspond to $\frac{1}{500}$ of this, or 40.008 megohms. Subtracting the resistance of the shunted galvanometer $20{,}000 \div 10 = 2{,}000$ ohms, or 0.002 megohm, leaves the unknown resistance 40.006 megohms. The algebraic equation expressing this is

$$X = \frac{(R + \frac{G}{m})dm}{d_1 m_1} - \frac{G}{m_1} = 40\,006 \text{ megohms}$$

Neglecting the resistance of the galvanometer in both cases, a simpler formula would give

$$X = \frac{R\,d\,m}{d_1\,m_1} = 40 \text{ megohms}$$

It may be noted that the difference between these results is an amount corresponding to a difference in deflection of 0.0075 of a single scale division, which is far smaller than the probable error which any observer would make. It is then clearly permissible to use the simpler formula,

$$X = \frac{R\,d\,m}{d_1\,m_1}$$

Example. In a high resistance measurement by the above method the known resistance was .2 megohms, and gave a deflection of 237 divisions, the multiplying power of the shunt being 100. With the unknown resistance inserted, the deflection was 178 divisions with the full current passing through the galvanometer. What was the value of this resistance? Ans. 26.6 megohms.

Voltmeter Method. Another method of measuring high resistance is that in which a sensitive high resistance voltmeter such as the Weston is used. This method, however, is not as accurate as the preceding and is not adapted to measurements of resistance greater than a few megohms. The voltmeter is connected in series with the unknown resistance and a source of constant e. m. f., as shown in Fig. 28. With such an arrangement the resistance X will be to the resistance of the voltmeter R, as the volts drop in X is to that in the voltmeter. The drop v in the voltmeter is given by its reading, and if the applied electromotive force V is known, the drop in X will be $V - v$. We therefore have the proportion,

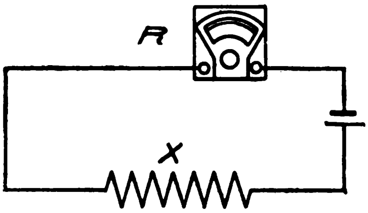

Fig. 28. Voltmeter Method of High Resistance Measurement.

$$X : R :: V - v : v, \text{ and}$$

$$X = \frac{V - v}{v} \times R$$

The voltage V, which should be at least 100, may be first determined by measurement with the voltmeter.

Example. A voltmeter having a resistance of 15,000 ohms, was connected in series with an unknown resistance. The e. m. f. applied to the circuit was 110 volts and the voltmeter indicated 6 volts. What was the value of the unknown resistance?

Solution. Applying the preceding formula

$$V = 110, v = 6, \text{ and } R = 15{,}000,$$

therefore

$$X = \frac{110-6}{6} \times 15{,}000 = 260{,}000 \text{ ohms, or } .26 \text{ megohms}$$

Ans. .26 megohms.

Insulation Resistance. The measurement of insulation resistance is performed by either of the two preceding methods of measuring high resistance. The voltmeter method is the simpler, but since it cannot be used to measure resistances greater than a few megohms, the direct deflection method proves to be the more valuable. The insulation of low potential circuits, however, need not exceed five megohms, and in testing such circuits the voltmeter method may be used. If little or no deflection is obtained it is then evident that the insulation is at least several megohms, which is all that is desired. As the insulation of high potential circuits must be greater than five or ten megohms, the direct deflection method should then be used.

The *connections* in testing the insulation of a circuit by these two methods are similar to those shown in Figs. 27 and 28, the resistance X being replaced by the insulation of the circuit. This is accomplished by connecting one wire to the line and the other to the ground such as, to a gas or water pipe. The insulation of the line from the earth is then included in the testing circuit; the current passing from the battery, or other source, through the voltmeter or galvanometer to the line, from the line through the insulation to the ground, and then to the battery.

The insulation of a dynamo, that is, the resistance between its conductors and its frame, is tested in a similar manner. This resistance should be at least one megohm for a 110-volt machine, but two megohms is to be preferred and is customary. This insulation is measured by connecting one wire to the frame and the other to the binding post, brushes, or commutator. The insulation is then included in the circuit. Insulation resistance decreases with increase of temperature so that this test of a machine should be made after a full load run of several hours.

The e. m. f. used should be constant and of one to two hundred volts value. Secondary batteries are the best for this purpose but silver chloride testing cells are much used. The resistance of dielectrics increases by continued action of the current and this property is known as *electrification*. For this reason the deflection should not be read until after a certain period of electrification—usually one minute. This action is quicker in some materials than in others, and is also greater at low than at high temperatures.

Insulation Resistance of Cables. In the preceding cases of insulation resistance only a part of the insulation is under electric strain. In the case of submarine cables and lead covered cables used on land, the whole of the insulation is subjected to the electric strain. To test the resistance of a waterproof insulation, the insulated wire or cable may be immersed in a tank of water. Care should be taken to leave enough of the cable out of the water so that surface leakage near the ends may not interfere with the test. For short lengths of cable the resistance of the wire inside the insulating material may be ignored. The resistance between the wire and a metal plate immersed in the tank is practically the resistance of the insulation. Fig. 27 shows the arrangement of the apparatus. As a cable takes a certain charge as a condenser when subjected to an e. m. f., it is necessary to protect the galvanometer, by a short-circuiting switch between the galvanometer terminals, during the rush of current on first closing the circuit. The switch box *S*, Fig. 27, has such a short-circuiting switch. This is important as otherwise the galvanometer may be injured. If the insulation resistance is not too high the direct deflection method above described may be used. If the insulation is excellent the deflection produced by the leakage should be very small and some other method must be used.

Charge and Recharge Method. An excellent method in such cases is the charge and recharge method. In this method. Fig. 27, the cable is first charged for several minutes, care being taken to short-circuit the galvanometer. The circuit is then opened for, say, one minute and the circuit closed again, the short-circuiting switch of the galvanometer meanwhile having been opened. While the circuit was open, a certain part of the charge leaked out and this is now replaced by an equal added charge. The galvanometer makes a sudden throw due to this added charge and after many oscillations comes to rest. If the relation between the added charge and the galvanometer throw is known, the quantity added may be computed, and the leakage current is equal to the added charge (equal to that lost) divided by the time in seconds for which the circuit was open. If the steady leakage produces a measurable deflection, account should be taken of this in estimating that part of the sudden throw produced on closing the circuit again. This correction we here suppose to be negligible. To find the relation between charge and throw, a condenser of known capacity (farads) is charged by a known e. m. f. and then discharged through the galvanometer. The charge equals the product of capacity and e. m. f. used. The charge divided by the throw produced, gives the constant of the instrument as a ballistic galvanometer. As condensers are rated in microfarads (millionths of farads) care must be taken to use the value in farads if the value in ohms insulation resistance is required. If the value in microfarads is used, the final result will come out in megohms. If an e. m. f. E_1 volts and capacity C microfarads produces a ballistic throw d_1, and if an e. m. f. of E_2 volts produces a throw of d_2 on closing the circuit through the insulation under test after the circuit has been open for t seconds, ignoring the resistance of other parts of the circuit, the insulation resistance is

$$X = \frac{E_2 \times d_1 \times t}{E_1 \times d_2 \times C} \text{ megohms}$$

Example. If E_1 is 1.44 volts, C is 0.5 microfarad, d_1 is 28.8 cm., E_2 is 100 volts, d_2 is 20 cm., and t_1 is 60 seconds, what is X?

Ans. 12,000 megohms.

As a rule the insulation resistance per mile is required. The longer the cable the more surface is exposed to leakage; consequently it is evident that the insulation resistance per mile is found by multiplying the resistance of the sample by its length in fractions of a mile. That is, a mile of cable would have, say, one-quarter as much insulation resistance as a quarter of a mile of cable.

In the case of lead covered cables, no tank is required, as connection may be made with the lead covering instead of the immersed metal plate before mentioned.

Resistance of Lines. Telegraph, telephone, and power transmission lines may be measured in place to best advantage if one or more additional lines are available between the terminals. If only one wire is available both ends may be connected to ground and the resistance, which involves that of the connections to ground and that of the earth return, may be measured by one of the methods described above. Such a method though unsatisfactory may be the best available. The resistance of the earth return is generally low, but there is always much uncertainty as to the resistance to earth at the ends. Earth currents of electricity, due to many causes, may much complicate the problem. When a second line of resistance X_2 is available, that and the unknown resistance X_1 may be connected together at the distant end and the combined resistance R, which equals X_1+X_2, measured. Next, the distant junction may be grounded and the two wires connected as the proportional arms of a Wheatstone's bridge, as illustrated in Fig. 29. The resistances in the other proportional arms are R_1 and R_2. One terminal of the battery is grounded. When a balance is obtained the proportion of the whole resistance R in X is

$$X_1 = R \frac{R_1}{R_1 + R_2}$$

In a similar way, $X_2 = R \dfrac{R_2}{R_1 + R_2}$

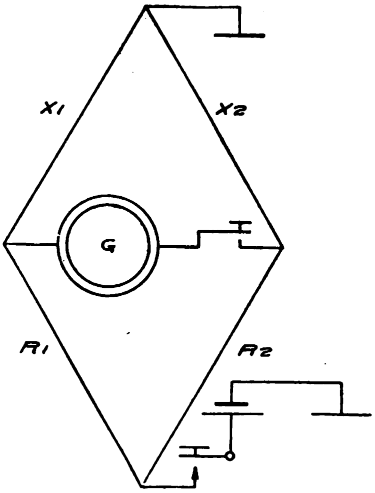

Fig. 29. Bridge Diagram for Line Resistance Measurement.

It is well to connect the battery in that branch of the bridge which includes the earth return as there may be a difference of potential due to earth currents, which does not disturb the *bridge* as it simply adds to or subtracts from the battery e. m. f. If a third line is available it may be used in the battery branch in place of the earth return. It will be noted that the resistance to earth at both ends is not in any of the four proportional arms and consequently does not affect the result.

Example. The two wires looped together have a resistance of 248 ohms. When a balance is obtained with them as arms of the bridge, $R_1 = 1{,}000$ ohms and $R_2 = 1{,}127$ ohms, the proportion being $R_1 : R_2 : : X_1 : X_2$. What are X_1 and X_2?

Ans. $X_1 = 116.6$ ohms; $X_2 = 131.4$ ohms.

Locating Grounds. In case a line wire is grounded at some unknown point, the above method may be used in locating the ground. The grounded wire and a second wire free from grounds may be looped together and their combined resistance measured. The loop, as before, is connected as two arms of a bridge, but the junction is left insulated. X_1 is now the resistance from the testing station to the point where the wire is grounded. X_2 is the combined resistance of the rest of that wire and the whole of the other wire. The resistance to the grounded point is then

$$X_1 = R \frac{R_1}{R_1 + R_2}$$

As a rule the resistance of every line is part of the office data, and therefore $R = X_1 + X_2$ is known in advance and need not be remeasured. As the resistance per mile is also usually part of the office data, the actual distance corresponding to X_1 may be computed and a lineman sent to the point to make the repair. If in a severe storm several grounds occur on the same wire, this method, of course, cannot be used to locate the trouble. In the case of ocean cables this method is used with excellent results. The cable repair steamer can be sent to the point of trouble where the cable is raised and repaired.

Locating Faults. This method may be used in the care of a broken submarine cable if both ends are exposed to the water, but it cannot be used for broken land-lines because the ends, even if both on the ground, are too imperfectly grounded. If the conductor of a submarine cable is broken but the insulation left intact, this method cannot be used. A method, however, in which the distributed capacity of the cable is measured in microfarads (see Capacity Measurements later) can be used to determine the location of the break. This latter method may also be used for a broken land-line where the end of the wire hangs free of the ground.

MEASUREMENT OF BATTERY RESISTANCE

Voltmeter Method. The following voltmeter method may be used to measure battery resistance. The battery of one or more cells is connected in circuit through a key K, with a known resistance R. The voltmeter of appropriate range is connected, as shown in Fig. 30, to the terminals of the battery. With the key K open, V_1—the e. m. f. of the battery—is measured. The key K is then closed and V_2—the reading of the voltmeter—is observed. By Ohm's law the current is $\frac{V_2}{R}$. A part of the battery's e. m. f., equal to $V_1 - V_2$, is now lost inside the battery because of the resistance X of the battery. We then have the relation,

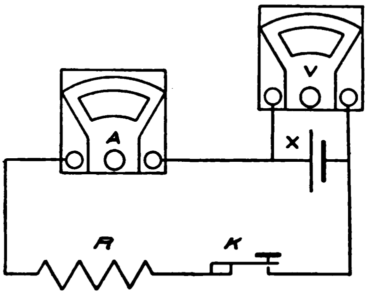

Fig. 30. Voltmeter Method of Measuring Battery Resistance.

$$V_1 - V_2 = X\frac{V_2}{R}, \text{ and}$$

$$X = R\,\frac{V_1 - V_2}{V_2}$$

If the resistance R is not known, an ammeter may be introduced into the circuit in series with R, and the current I measured directly. Then

$$XI = V_1 - V_2, \text{ and}$$

$$X = \frac{V_1 - V_2}{I}$$

It will be noticed that it is tacitly assumed that when the key K is open, not enough current will pass through the voltmeter to introduce any error. If the battery resistance is large this error is not negligible and a sensitive high resistance galvanometer with considerable additional resistance, perhaps 100,000 ohms besides, may be substituted for the voltmeter. If the deflections of the galvanometer in the two cases (open and closed) are d_1 and d_2, we then have

$$X = R\,\frac{d_1 - d_2}{d_2}$$

As the battery when furnishing a current begins at once to fall off in e. m. f., that is, *polarize*, a small error due to polarization makes the battery resistance appear too high. Such a value of the resistance R should be chosen as to make the deflections materially different. Otherwise a slight error in V_1 and V_2 or d_1 and d_2 will make their difference $V_1 - V_2$ or $d_1 - d_2$ many per cent in error.

Example. A cell has an e. m. f., $V_1 = 1.47$ volts when S is open and 1.12 when S is closed. R is 5 ohms. What is X?

Ans. $X = 1.56$ ohms.

Mance's Method. Another method is Mance's method, in which the battery, whose resistance X is to be determined, forms one arm of a Wheatstone's bridge, as indicated in Fig. 31. No key is placed in the galvanometer branch and no additional resistance, in the branch which includes the key S. The resistance in R_3 is adjusted until the galvanometer does not change its deflection on closing the key S. It is usually necessary to put considerable additional resistance R in the galvanometer arm to keep the deflection small. If the deflection does not change on closing the key S, it is evident that the decrease in the potential difference at the terminals of the cell due to its increased current when S is closed, must exactly equal the decrease in the potential difference between the terminals of R_3 due to this path being robbed of a part of its current because of the new path. Otherwise the potential difference at the galvanometer terminals, which is the difference of the potential differences over the two arms, would change and the deflection change. Similar reasoning applies to R_1 and R_2, only here the difference over R_2 increases by just the amount that that over R_1 falls, thus keeping the total amount constant for the combination of R_1 and R_2. The arrows show the direction of the currents in the various arms. If no change in the galvanometer current occurs, the changes in R_1 and R_3 must be equal and so also the changes in R_2 and X. It follows then if the galvanometer deflection remains constant whether S is open or closed, that

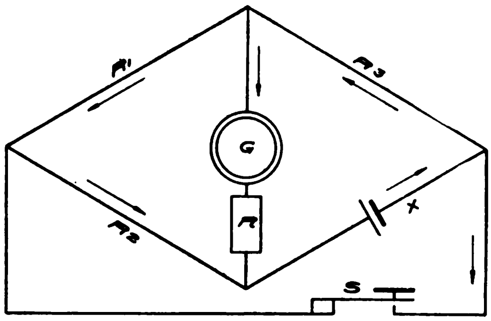

Fig. 31. Bridge Diagram for Mance's Method.

$$R_1 : R_2 :: R_3 : X, \text{ or}$$

$$X = \frac{R_2 \times R_3}{R_1}$$

Should the battery polarization change on closing the key S, the galvanometer deflection will change. For this reason the key S should be closed for an instant only.

Besides these methods there are excellent methods in which alternating currents are used, but they are too advanced to be described in this course. Such alternating-current methods should be used in measurement of the resistance of solutions (so called electrolytes) which are decomposed by a direct current.

MEASUREMENT OF ELECTROMOTIVE FORCE

Voltmeter Method. The simplest method of measuring an electromotive force is by the use of a voltmeter which indicates directly the number of volts. A voltmeter of the proper range should be chosen. For very small e. m. f 's the millivoltmeter, usual range 1 to 300 millivolts, *i. e.*, 1 to 0.3 volt, may be used. For higher e. m. f.'s, voltmeters reading to 1.5, 15, 150, and 300 volts respectively; are made by the Weston Electrical Instrument Co. and others. A voltmeter

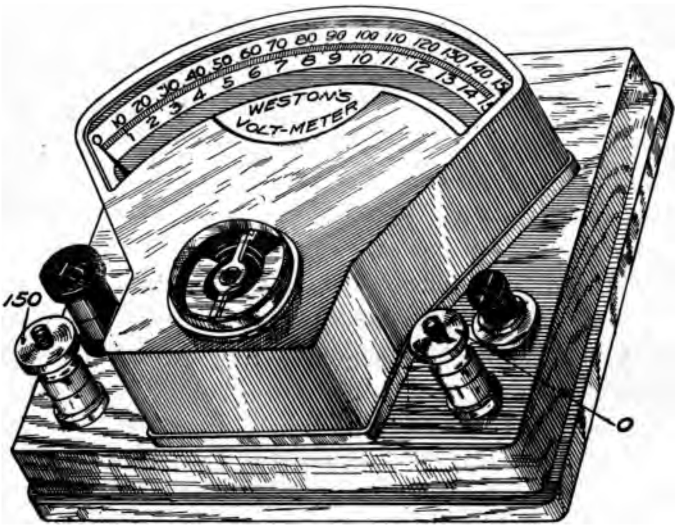

Fig. 32. Commercial Portable Voltmeter.

is simply a galvanometer calibrated to be read in volts. It is evident that if an additional resistance equal to that of the voltmeter is placed in series with the latter, twice the voltage will be required to produce the same deflection as before. In general, any resistance in series which makes the total resistance *n* times that of the voltmeter alone, may be used as a multiplier; and the reading of the voltmeter must be multiplied by *n* to get the value of the e. m. f. Such a multiplier may be bought with a voltmeter in order to make its effective range greater. For example, if the resistance of a voltmeter of range 0 to 150 volts, is 15,000 ohms, a multiplier having a resistance of 60,000 ohms will bring the total up to 75,000 ohms, and the constant *n* of the multiplier is 5. With this multiplier in the circuit the upper limit of the voltmeter is extended to 750 volts. If the multiplier is mounted inside the voltmeter, and if on using the binding posts marked 0 and 15 the range is 0 to 15 volts, and using the binding posts marked 0 and 150 the range is 0 to 150 volts, the multiplier evidently must have nine times the resistance of the main part. Such a voltmeter, shown in Fig. 32, is said to be a two-scale voltmeter, and it may be used equally well on either range. As the extra expense of providing the multiplier and extra binding post is slight, a two-scale voltmeter is a very inexpensive substitute for two voltmeters. It is also evident that a low range voltmeter may be used in connection with any resistance box as a multiplier. The Weston voltmeters have approximately 100 ohms resistance per volt of range and, therefore, take a maximum of about 0.01 amperes when used on an e. m. f. which is the maximum of the range. As a rule the current taken by a voltmeter is negligible in comparison with the current in the rest of the circuit.

The voltmeter method may be used equally well with both direct and alternating electromotive forces.

Potentiometer Method. For the comparison of e. m. f.'s, the potentiometer is the most accurate apparatus. When a balance is reached the e. m. f.'s to be compared are not allowed to furnish any current, and consequently no polarization results in their source. The effect of internal resistance is absolutely *nil* also. The arrangement of apparatus is shown in Fig. 33.

Two resistance boxes *M* and *N*, each of 10,000 ohms capacity, are arranged to have plugs withdrawn to a total of 10,000 ohms, and are connected in series with a battery *B*. To avoid injuring *B*, plugs corresponding to 10,000 ohms should be withdrawn before connecting it in circuit. The circuit has a high resistance and the effect of polarization of the battery B quickly reaches its limit and a steady current I flows through the circuit. If the resistance in box M is R ohms, that in box N is (10,000 − R) ohms; and the fall of potential over M is $R\,I$ volts, and over the box N is (10,000 − R) I volts. One of the cells to be compared, a standard cell of e. m. f. S, is connected in series with some high resistance A, a sensitive galvanometer G, and a key K. In general, on closing the key K the galvanometer will deflect; but if the resistances in M and N are adjusted until the potential difference over M is exactly equal to the e. m. f. of the cell S, the latter is in perfect balance and can neither supply current to the general circuit supplied by B nor can current be forced backward through the cell S. In that case the e. m. f. of S equals the fall of potential through M, and, calling R_1 the resistance in M,

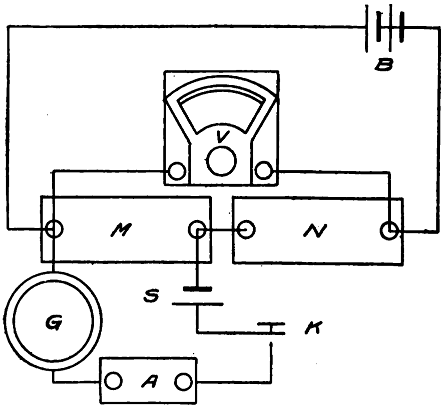

Fig. 33. Potentiometer Method of Measurement of E. M. F.

$$S = I\,R_1$$

If now a second cell of unknown e. m. f. X is substitued for S, and the resistances in M and N readjusted—but their sum kept 10,000 —until on closing the key K no deflection results, calling the new value of the resistance in M, R_2, we have the relation

$$X = I\,R_2$$

It follows that

$$X = S\,\frac{R_2}{R_1}$$

If S is known, X may be computed. It is well to repeat the balance with S to be quite sure that no change has meanwhile occurred in the main battery B. Precaution should be taken in setting up the apparatus that B is greater than either S or X and that they are connected into the circuit so that they are in opposition to B. If these conditions are not obeyed, it is evident that no balance can be obtained. If no exact balance can be obtained, but a change of one ohm changes the galvanometer deflection from up to down the scale, the fraction of an ohm needed for an exact balance can be obtained by interpolation. In the next article on the calibration of a voltmeter such an interpolation is made.

Example. With a total resistance in M and N of 10,000 ohms of which the resistance in M was $R_1 = 5{,}267$ ohms, when the standard Clark cell of 1.433 volts was in the galvanometer circuit, and the resistance was $R_2 = 5{,}470$ ohms when the Leclanché cell was in the galvanometer circuit, what is the e. m. f. X of the Leclanché cell?

Ans. $X = 1.4882$ volts.

Calibration of a Voltmeter. If during the previous experiment, a voltmeter had been connected for the whole time between the extreme terminals of M and N, the potential difference V between the terminals of the voltmeter would have been

$$V = 10{,}000\, I = 10{,}000 \frac{S}{R_1}$$

This gives a convenient method of calibrating a voltmeter by means of a standard cell of known e. m. f. S. The calibration may be extended to various points of the voltmeter by changing the e. m. f. of the main battery B. In such cases the resistance in M, to obtain a balance, will change in the inverse ratio. It is evident that the calibration cannot by this method be extended to points below the e. m. f. of the standard cell. In the case of high e. m. f.'s, it is desirable to increase the total resistance in M and N beyond 10,000 ohms. For example, if the e. m. f. produced by B at the terminals of the voltmeter is 150 volts, a total resistance of 100,000 ohms would be about right. In this case over 99,000 ohms would be in N and less than 1,000 in M. If several standard cells are available, they may be connected in series in the galvanometer branch, thus increasing the resistance for a balance in the box M. As most boxes have one ohm for their smallest resistance, a greater per cent of accuracy is obtained if the resistance in M is large. If an exact balance cannot be obtained and the nearest smaller resistance produces a deflection d_1 one way, and the nearest larger resistance produces a deflection d_2 in the opposite direction, the fraction of an ohm which would have produced a balance is evidently

$$\frac{d_1}{d_1 + d_2}$$

Example. With a total resistance of 100,000 ohms in M and N, and 987 ohms in M producing a deflection of 5 divisions down the scale, and 988 ohms producing a deflection of 15 divisions up the scale, and a standard Clark cell of 1.433 volts e. m. f. in the galvanometer circuit, what is the correction to be added to the voltmeter reading which was 144.9 volts? Ans. Correction = + 0.25 volt.

Suggestion of Solution. The change in deflection by change of one ohm is 5 + 15 scale divisions; therefore, 987 is 0.25 ohm too small and 988 is 0.75 ohm too large. The e. m. f. figures out 145.15 volts; therefore, 0.25 volt will be added to the voltmeter reading to give the correct result.

This method, as will be seen later under the head of "Measurement of Current," can be used with a standard cell to measure a current.

Condenser Method. If a condenser of capacity C is connected by means of a charge and discharge key K, which has an upper and a lower contact, as shown in Fig. 34, alternately to a standard cell of e. m. f. B, and a ballistic galvanometer G, the throw d of the galvanometer will be a measure of the charge of the condenser equal to $B \times C$. If, now, a cell of unknown e. m. f. X is substituted for B, the deflection d_2 will be a measure of the charge of the condenser now equal to $X \times C$. It follows that

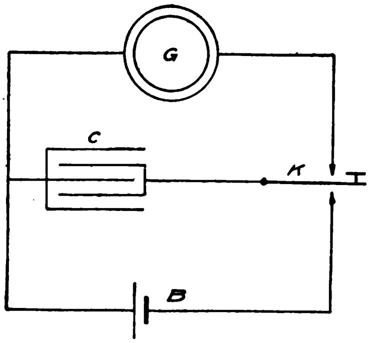

Fig. 34. Condenser Method of Measurement of E. M. F.

$$X = B\frac{d_2}{d_1}$$

This method is free from difficulties due to polarization and internal resistance of the cells; because the very small charge taken by the condenser produces no measurable polarization, and the effect of internal resistance is only to lengthen the time of charging of the condenser, but not to change the total quantity.

The accuracy of the method, however, is limited to that of the reading of the deflections d_1 and d_2, and it is difficult to get results much closer than, say, $\frac{1}{5}$ of 1%. The potentiometer method will give results easily to $\frac{1}{1000}$ of 1% if the resistances used are accurate; and in general the accuracy of the potentiometer method is limited only by the accuracy of the resistances. For accurate comparisons the potentiometer method is always used.

VOLTAIC CELLS AND BATTERIES

A *voltaic cell* is usually composed of a pair of electrodes immersed in a liquid or in two liquids separated from one another by a porous partition. The liquid, or liquids, must be what is known as an *electrolyte;* and must undergo a chemical breaking up, called *electrolysis,* with chemical action on one or both electrodes when the circuit is closed and a current flows through the cell.

Some cells fall rapidly in e. m. f. when the circuit is kept closed. This phenomenon is known as *polarization,* and it frequently is due in large part to the deposit of a film of hydrogen gas on the surface of one of the electrodes (the cathode). This gas is one of the products of the electrolysis of the liquid. If the cathode, for example a copper plate, as in the case of the gravity cell, is surrounded by a solution of copper sulphate, the hydrogen does not reach the copper plate but is intercepted by the copper sulphate solution, and copper instead of hydrogen is deposited. Naturally, the deposit of copper on a copper plate produces no polarization. The copper sulphate solution is called a *depolarizer.* The other electrode of the gravity cell is zinc and is immersed in a dilute solution of either zinc sulphate or sulphuric acid.

In the *Grove* and the *Bunsen* cells, the depolarizer is nitric acid in a porous cup in which the cathode is immersed. The nitric acid is rich in oxygen which it gives up to oxidize the hydrogen gas, thus forming water (H_2O) which makes the solution more dilute but causes no polarization. Cells with liquid depolarizers cannot be left on open circuit as the liquids diffuse into one another and the cell is spoiled. Cells for open circuit use must have either a solid or a paste for a depolarizer. The Leclanché cell has manganese dioxide (a solid) packed in a porous cup about its cathode of carbon. The data of various cells can be found in books on cells.

Standard Cells. There have been various cells used as standards of e. m. f. of which the *Clark* and the *Weston* have received most attention. A *standard cell* must be composed of materials which, while the cell is in use, do not change; that is, no new substance may be formed by the action of the cell; the cell should have little or no polarization when used by zero methods like the potentiometer method described above; the cell also must not deteriorate when left on open circuit.

Both the Clark and the Weston cells fulfill these requirements. At the International Electrical Congress, held in Chicago, 1893, the normal Clark cell was recommended for international legalization and its e. m. f. at 15° C. was voted to be considered for practical purposes as 1.434 volts; and a committee, consisting of Professors von Helmholtz, Ayrton, and Carhart, was charged with the duty of drawing up specifications for the precise form of the cell. Von Helmholtz died soon afterward and the other members of the committee could not agree on the specifications, with the result that the principal countries (in electrical matters) have never agreed on a form for the cell.

It has now been displaced by the Weston cell, which, in 1908, was recommended by an International Conference in London as an international standard. It is now known that the Clark cell has an e. m. f. slightly below 1.433 volts, instead of 1.434 volts as thought in 1893. The normal Clark cell uses as materials zinc amalgam in a saturated aqueous solution of zinc sulphate, with an excess of zinc sulphate crystals present, and pure mercury in the presence of mercurous sulphate in the form of a paste which acts as the depolarizer. The action of the cell is to form more zinc sulphate and reduce some of the mercurous sulphate to mercury, or *vice versâ*, when the current flows in the direction of the e. m. f. or is driven in the opposite direction by a greater outside e. m. f. The principal objection to the normal Clark cell is that its e. m. f. changes by a considerable amount with change in temperature, falling about 0.08% for every Centigrade degree rise in temperature.

The Weston normal cell is similar to the Clark cell except that cadmium replaces the zinc, and cadmium sulphate the zinc sulphate The Weston normal cell has a much lower temperature coefficient than the Clark cell, its e. m. f. falling about 0.00406% for each Centigrade degree rise above 20° C. and *vice versâ*. The e. m. f. of the

Weston normal cell at 20° C. was recommended by the London conference of 1908 to be taken provisionally as 1.0184 volts.

Storage Cells. Many voltaic cells when exhausted, may be recharged by forcing current through the cell in the reverse direction by the application of an outside e. m. f. greater than the e. m. f. of the cell. Such cells are called *storage cells*. In general only such cells as form no *new* kind of material when discharging are reversible, and evidently a cell to be charged and discharged repeatedly must be reversible.

All standard cells must be reversible. Reversibility, however, is not the only requirement of storage cells to be used commercially. Other qualities required are low internal resistance, large capacity for charge measured in ampere hours in comparison with size and weight, long life under service, ability to stand without harm in open circuit, moderate cost, etc.

The storage cell most used commercially has both plates of lead with dilute sulphuric acid as electrolyte and lead peroxide as the depolarizer. The lead peroxide is a solid or paste which adheres to the positive pole of the battery. The e. m. f. of a lead cell is about 2.2 volts when fully charged and may safely be discharged until its e. m. f. is reduced to 1.8 volts. When the cell is charged one plate has a deposit of lead peroxide, and the other has a spongy texture, due to its reduction from an oxide or a sulphate of lead in its previous history. When the battery is discharged, the sulphuric acid is electrolyzed; the hydrogen formed reduces some of the lead peroxide of the positive, and the sulphion forms some insoluble lead sulphate from the negative. The sulphuric acid becomes more dilute. On recharging the cell the lead sulphate is reduced to spongy lead at the negative, some additional lead peroxide is formed on the positive, and the density of the sulphuric acid increases. For details as to the manufacture of the various varieties of lead cells and other storage cells, the reader is referred to works on storage cells.

To increase the capacity of a cell the negative consists generally of a number of plates connected together both electrically and mechanically, and the positive consists of one plate less in number and connected together in the same manner. The positive plates are interlarded between the negative plates.

Storage cells are also called *secondary cells* or *accumulators* by some writers.

Batteries. The word battery is technically used to mean a group of cells. In common parlance the word is used sometimes to mean a single cell; but this use is not to be recommended.

MEASUREMENT OF CURRENT

Electrodynamometers. In the choice of a unit of current, it was decided that a unit of current in a straight conductor at right angles to a unit magnetic field, should exert a force (at right angles to both the directions of current and field) of one dyne per unit length of the conductor. As mentioned earlier, this unit of current was thought to be inconveniently large by the committee of the British Association for the Advancement of Science, which had the matter of electrical units in charge, and as a consequence for practical purposes they recommended that one-tenth of this theoretical unit should be taken as the practical unit. The latter unit is called the *ampere.* As the magnetic field due to the flow of an electric current in coils, may be computed from the data of the coils and the current, it is evident that absolute electrodynamometers may be made to measure current without the intervention of other electrical measuring apparatus. These absolute electrodynamometers may take various forms, including that of current balance. By means of an absolute electrodynamometer and a standard resistance, the e. m. f. of standard cells may be determined with a high degree of accuracy according to the principle of Ohm's law.

By comparison either directly with the standard electrodynamometer or indirectly by means of standard resistances and standard cells, other forms of electrodynamometers and all forms of ammeters may be adjusted so as to read amperes. It is evident from the above that as more improved absolute electrodynamometers are constructed, we may expect greater exactness in the determination of the e. m. f. of the Weston normal cell, which for the present is taken as 1.0184 volts at 20° C.

Ammeters. An ammeter is a galvanometer graduated so that it reads current directly in amperes. This graduation is obtained directly by comparison with either an absolute electrodynamometer or indirectly by means of a standard cell and standard resistances.

Ammeters for any desired range of current are on the market, and the accuracy of their readings is in proportion to the care with which they have been constructed and calibrated. Even the best are moderate in price and the poorest should not be one per cent in error.

Ammeters to measure large currents carry the main portion of the current through shunts which differ theoretically in no respect from the shunts used with other forms of galvanometer.

To measure current by means of an ammeter involves introducing the instrument into the circuit, care being taken to connect the terminal marked + to the positive terminal of the source of e. m. f. The exact position of the ammeter in an undivided circuit is not important, as the current is the same throughout the circuit. Care should be taken that the ammeter has a range which the current does not exceed; otherwise the pointer may be bent or even the ammeter may be burned out by the action of excessive current. An ammeter has an exceedingly small resistance, and to connect an ammeter without additional resistance between the terminals of a dynamo or a battery is to produce a short circuit practically. Excessive current will flow through the ammeter and it probably will be destroyed.

Ammeters designed for direct-current circuits cannot be used on alternating-current circuits. Some forms of A. C. ammeters may be used on D. C. circuits; but as a rule ammeters should be used on the type of circuit for which they are designed.

Calibration of Ammeters. An ammeter may be compared directly with another in the same circuit by putting them in series and observing their readings with various values of the current. The current may be varied by changing the resistance in the circuit or the e. m. f. The potentiometer method may be used to calibrate an ammeter.

Potentiometer Method. The most exact method of measuring a current, assuming that the e. m. f. of a standard cell and the resistance of standard coils are known, is the potentiometer method. This is not an *absolute* method. The arrangement of the apparatus is somewhat complicated; but if it is compared with the potentiometer method as used for the comparison of e. m. f.'s (page 41) it will be seen that the commutator C, Fig. 35, is used to insert in the galvanometer branch either the standard cell of known e. m. f. S or a potential difference over a known resistance R due to the current to be measured I. The current I may be passed also through an ammeter A in circuit with R, a rheostat and an auxiliary battery B_2 which causes the current I to flow through the circuit. In the lower part of the diagram the galvanometer G and, if desired, a high resistance to protect the galvanometer, are connected between the left end of the commutator and one terminal of the resistance box L; the right end of the commutator is connected to the other terminal of L. The auxiliary battery B_1 serves to send a constant current through L and M, whose combined resistance is kept constant at, say, 10,000 ohms. In setting up the apparatus, 10,000 ohms should be inserted in L and M before connecting the battery B_1; otherwise the battery will become badly polarized due to an excessive current. This precaution is very important. When a balance is obtained no current passes through the galvanometer on closing the key K.

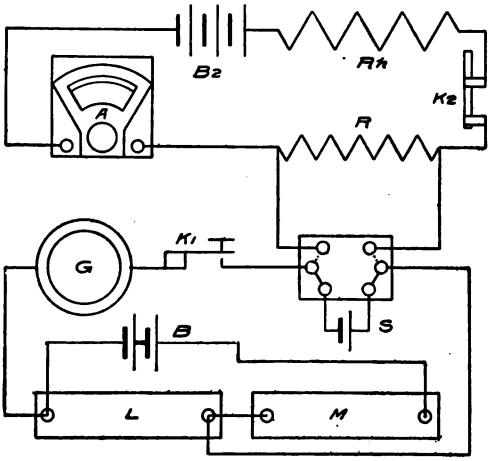

Fig. 35. Potentiometer Method of Current Measurement.

The order of procedure is as follows: first, connect S in circuit by means of the commutator C and shift resistance from L to M, or *vice versâ* until, on closing the key K a balance is reached, as indicated by a zero deflection of the galvanometer. The fall of potential through L, which under these conditions is exactly equal to the e. m. f. of the standard cell S, is then equal to its resistance R_1 multiplied by the current I from the battery B_1, or $S = R_1 I_1$; second, close the upper circuit by K_2, reverse the commutator to the position shown in the figure by dotted lines, which throws the p. d. over R into the galvanometer circuit. Adjust L and M until no current flows through the galvanometer on closing the key K_1. The total resistance in L and M is still kept 10,000 ohms. The resistance in L now has a value R_2, and the potential difference over L is now $R_2 I_1$ and is also in the galvanometer circuit. If no current flows through the galvanometer, $R_2 I_1$ must exactly equal and oppose the fall of potential $R I$ over the resistance R. We then have

$$S = R_1 I_1, \quad \text{and } R I = R_2 I_1,$$

or, combining these equations,

$$I = \frac{R_2 \times S}{R \times R_1}$$

By adjusting the rheostat in the upper, or the battery B_2 circuit, the current to be measured I, and consequently the point at which the calibration of the ammeter is desired, may be changed at will. The only limitation is that the battery B_1 must have a higher e. m. f. than the standard cell's e. m. f., and also higher than the largest p. d. over the resistance R.

As the e. m. f. of B_1 may be increased, if necessary, by introducing additional cells into B_1, there is no limit to the current I (in the upper circuit) which may be measured. In the case of very large currents the standard resistance R should be of relatively low resistance. If too much heat is developed in R its temperature may rise with consequent change of resistance.

Standard resistances are made of manganin which will carry any reasonable current without undue heating or change of resistance. These standards are arranged so that the wire may be immersed in an oil bath (pure petroleum) which may be kept stirred so that the heat may be carried away. A thermometer in the oil bath may be used to measure the temperature. Allowance thus may be made for any change in resistance due to change in temperature.

Such standards are made by the best manufacturers for 100, 10, 1, 0.1, 0.01, and 0.001 ohms resistance. Standard resistances calibrated by the Bureau of Standards at Washington are moderately expensive, but the cost is not higher than the cost of manufacture and testing at the Bureau would warrant. The certificate which accompanies each coil states the resistance at 20° C. and the change of resistance per degree change of temperature.

Silver Voltameter Method. If a current is passed through a solution of silver nitrate, each ampere deposits 0.001118 gram of silver from the solution each second. This is closely equal to $4 \frac{1}{40}$ grams per hour. The silver voltameter is very difficult to handle with the degree of accuracy that is necessary for good results, so it is not recommended for general use.

To set up a voltameter, a platinum bowl is used as a cathode and a plate of pure silver as an anode. The electrolyte, 15 to 20 parts by weight of pure silver nitrate added to 100 parts of distilled water, is placed in the bowl, and the anode immersed in the solution. A current density of $\frac{1}{5}$ ampere or less per sq. cm. is allowed at the anode, and of $\frac{1}{50}$ ampere or less per sq. cm. at the cathode. Care is to be taken that no particles of silver mechanically detached from the anode shall reach the cathode. This may be accomplished by wrapping the anode in clean filter paper.

Before weighing the cathode to determine its increase in weight, any trace of the solution must be removed by careful working with distilled water and the cathode dried. This seems easy, but it is difficult, in fact.

The solution should be made anew for each experiment.

EXAMINATION PAPER

ELECTRICAL MEASUREMENTS

PART I

Read Carefully: Place your name and full address at the head of the paper. Any cheap, light paper like the sample previously sent you may be used. Do not crowd your work, but arrange neatly and legibly. *Do not copy the answers from the Instruction Paper; use your own words, so that we may be sure that you understand the subject.*

1. What is the distinction to be made between fundamental and derived units? Give examples of each.

2. Describe briefly the different types of galvanometers and explain wherein they differ and the advantages of each.

3. Voltmeters and ammeters are really *galvanometers.* Why do they fall into this class and to which type do they belong?

4. Explain the *lamp and scale* and the *telescope and scale* methods of reading galvanometer deflections.

5. Describe the control magnet as used with needle galvanometers and explain its function.

6. Describe and explain the electrodynamometer and the wattmeter. How do they differ?

7. Describe the rheostat. What materials may be used for the resistance?

8. How do resistance coils differ from the rheostat mentioned in Question 6? What material is generally used for accurate resistance units and why?

9. Describe and explain the use of shunts for galvanometers.

10. Explain the Wheatstone's bridge. Describe the two usual forms of the bridge.

11. Make the usual "diamond" diagram of the connections of a bridge and find the value of X when $M = 1{,}000$, $N = 10$, and $P = 3{,}247$.

12. Describe a good method for measurement of a low resistance.

13. Describe good methods for measurement of a fairly high resistance.

14. How do you measure a *very* high resistance?

15. Describe the method for finding the place where a telephone line is grounded.

16. Describe a good method of measuring the resistance of a cell.

17. Describe the potentiometer method of measuring the e. m. f. of a cell. Why is this method best?

18. What is a standard cell and what are the requisites of such a cell?

19. Describe the Weston standard cell. Does its e. m. f. rise or fall with rise of temperature, and how much per degree Centigrade?

20. Describe the lead storage cell. What is its e. m. f. when charged, and when discharged? What materials are used for the positive and the negative plates and for the solution?

After completing the work, add and sign the following statement:

I hereby certify that the above work is entirely my own.

(Signed)